

E2.3 Algebraic fractions

Name: _____ Class: _____ Date: _____ **Total: 15 marks**

KEY WORDS algebraic fraction 代数分式 • factorise 因式分解 • common denominator 公分母
• difference of two squares 平方差 • simplify 化简

Objective

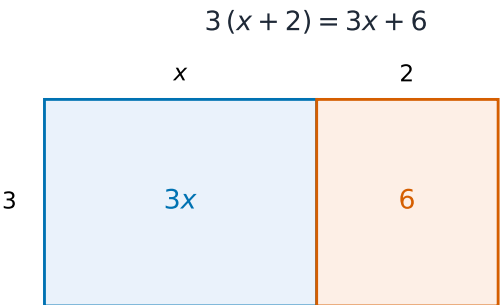
Build the skills to answer Extended exam questions on **algebraic fractions** 代数分式 — **simplifying** 化简 by **factorising** 因式分解 first, and adding or subtracting two fractions over a **common denominator** 公分母.

You must be able to:

- simplify an algebraic fraction by factorising the numerator and the denominator
- add or subtract algebraic fractions, writing the result as a single fraction
- cancel only a whole factor, never a single term

1 Worked examples

■ Factorise first — only a whole bracket can cancel



Factorising turns a sum into a product, and only products cancel

- **Simplify:** $\frac{x^2 - 9}{x^2 + 7x + 12} = \frac{(x - 3)(x + 3)}{(x + 3)(x + 4)} = \frac{x - 3}{x + 4}$ — the bracket $(x + 3)$ cancels

because it is a **factor** of both.

- **Never cancel a term.** In $\frac{x + 3}{x + 4}$ the x 's do **not** cancel; they are added, not multiplied.
- **Adding:** use the product of the denominators. $\frac{3x}{4} + \frac{x}{6} = \frac{9x + 2x}{12} = \frac{11x}{12}$.
- **Subtracting:** put the whole second numerator in a bracket, then subtract — $-3(x + 1)$ is $-3x - 3$, not $-3x + 3$.

Answer shapes (marked by **method marks** — M1 for a correct method, A1 for the correct answer): each correct factorisation, and the single combined fraction before simplifying, earns a method mark.

2 Practice

2.1 Simplify $\frac{x^2 - 25}{x^2 - 2x - 15}$. [3]

2.2 Write $\frac{x}{3} + \frac{2x}{5}$ as a single fraction. [2]

2.3 Simplify $\frac{6x^2y}{9xy^3}$. [2]

3 Exam-style questions

3.1 Write $\frac{3}{x-2} + \frac{2}{x+1}$ as a single fraction in its simplest form. [4]

3.2 Simplify $\frac{2x^2 + 5x - 3}{4x^2 - 1}$. [4]

Solutions

Marking: **M1** a correct method, **A1** the correct answer.

Each answer shows the steps, then the **result**.

2.1

- numerator is a difference of two squares: $x^2 - 25 = (x - 5)(x + 5)$
- denominator factorises: $x^2 - 2x - 15 = (x - 5)(x + 3)$

$$= \frac{\mathbf{x + 5}}{\mathbf{x + 3}}$$

2.2

- common denominator 15: $\frac{5x}{15} + \frac{6x}{15}$

$$= \frac{\mathbf{11x}}{\mathbf{15}}$$

2.3

- divide the numbers by 3 and cancel one x and one y

$$= \frac{\mathbf{2x}}{\mathbf{3y^2}}$$

3.1

- common denominator $(x - 2)(x + 1)$
- $\frac{3(x + 1) + 2(x - 2)}{(x - 2)(x + 1)}$
- expand the numerator: $3x + 3 + 2x - 4$

$$= \frac{\mathbf{5x - 1}}{\mathbf{(x - 2)(x + 1)}}$$

3.2

- numerator: $2x^2 + 5x - 3 = (2x - 1)(x + 3)$
- denominator is a difference of two squares: $4x^2 - 1 = (2x - 1)(2x + 1)$

$$= \frac{\mathbf{x + 3}}{\mathbf{2x + 1}}$$