IGCSE MATHEMATICS • EXERCISE SHEET

E2.2 Algebraic manipulation

Name: _____ Class: _____ Date: _____ **Total: 25 marks**

KEY WORDS term 项 • expand 展开 • area 面积 • difference of two squares 平方差 • factorising 因式分解

Objective

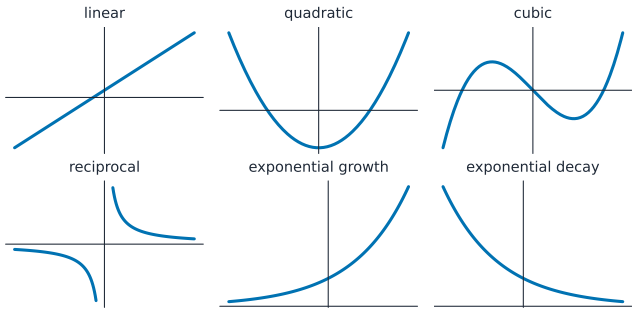
Build the skills to answer Extended exam questions on **algebraic manipulation** 代数运算 — **expanding brackets** 展开括号 and **factorising** 因式分解 (common factors, difference of two squares, quadratics).

You must be able to:

- expand single and double brackets
- factorise by taking out a common factor
- factorise a difference of two squares and a quadratic

1 Worked examples

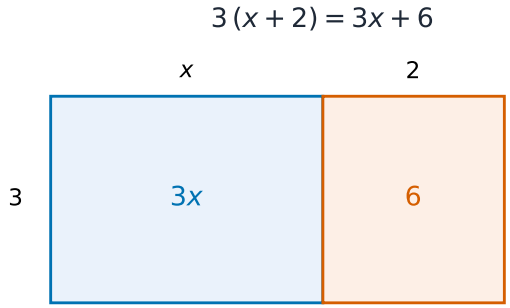
Expand and factorise



recognise $y = x^2$, $y = 1/x$, $y = 2^x$ from shape

Factorising is the reverse of expanding

- **Expand:** $(2x + 1)(x - 4) = 2x^2 - 7x - 4$.



$a(b + c) = ab + ac$ · area model of expansion

The area model of expanding a bracket: $3(x + 2) = 3x + 6$

- **Common factor:** $9x^2 + 15xy = 3x(3x + 5y)$.
- **Difference of two squares** 平方差: $9x^2 - 16 = (3x + 4)(3x - 4)$.
- **Quadratic:** $2x^2 + 7x + 3 = (x + 3)(2x + 1)$ (split the middle term).

Answer shapes: M1 a correct method, **A1** the correct answer.

Choosing the factorising method

Work through these in order; the first one that applies is the right one.

1. **Common factor first, every time.** $6x^2 + 9x = 3x(2x + 3)$. Missing it makes everything afterwards harder, and the mark for the common factor is lost even if the rest is right.
2. **Difference of two squares**, when there are two terms and both are squares with a minus between: $a^2 - b^2 = (a + b)(a - b)$. So $x^2 - 49 = (x + 7)(x - 7)$ and $9x^2 - 25 = (3x + 5)(3x - 5)$.
3. **A quadratic with $a = 1$:** find two numbers that multiply to the constant and add to the coefficient of x. For $x^2 + 3x - 10$ they are $+5$ and -2, so it factorises as $(x + 5)(x - 2)$.
4. **A quadratic with $a \neq 1$:** multiply a by c, find two numbers multiplying to that and adding to b, split the middle term and factorise in pairs.
5. **Four terms:** factorise in pairs, then take out the common bracket: $xy + 2x + 3y + 6 = x(y + 2) + 3(y + 2) = (x + 3)(y + 2)$.

Worked example (the $a \neq 1$ method). Factorise $2x^2 + 5x - 3$.

1. $a \times c = 2 \times (-3) = -6$.
2. Two numbers multiplying to -6 and adding to $+5$: $+6$ and -1.
3. Split the middle term: $2x^2 + 6x - x - 3$.

4. Factorise in pairs: $2x(x+3) - 1(x+3)$.
5. Take out the common bracket: $(2x-1)(x+3)$.

Always **check by expanding**. It takes one line and catches a sign error, which is the commonest loss on this topic.

2 Practice

2.1 Expand $3x(2x-5)$. [1]

2.2 Factorise $6a + 9ab$. [1]

2.3 Expand and simplify $(x+2)(x-5)$. [2]

2.4 Factorise fully $12x^2 - 27$. [2]

2.5 Factorise $ab + 4a + 3b + 12$. [2]

3 Exam-style questions

3.1 Factorise $x^2 - 49$. [1]

3.2 Factorise $x^2 + 3x - 10$. [2]

3.3 Factorise $2x^2 + 5x - 3$. [2]

3.4 (a) Factorise fully $18x^2 - 8$. [3]

(b) Factorise $3x^2 - 11x + 6$. [3]

(c) Simplify $\dfrac{x^2-9}{x^2+x-12}$, showing your factorising. [3]

(d) Hence solve $\dfrac{x^2-9}{x^2+x-12} = 2$. [3]

Solutions

Marking: **M1** a correct method, **A1** the correct answer.

2.1 $3x(2x-5) = 6x^2 - 15x$.

2.2 $6a + 9ab = 3a(2+3b)$.

2.3 $(x+2)(x-5) = x^2 - 5x + 2x - 10 = x^2 - 3x - 10$.

2.4 Common factor 3: $3(4x^2-9)$; difference of two squares: $3(2x+3)(2x-3)$.

2.5 $a(b+4) + 3(b+4)$; $(a+3)(b+4)$.

3.1 $x^2 - 49 = (x+7)(x-7)$.

3.2 two numbers multiplying to -10 and adding to 3 are 5 and -2: $(x+5)(x-2)$.

3.3 $a \times c = -6$, middle $+5$: numbers 6 and $-1 \rightarrow 2x^2+6x-x-3 = 2x(x+3)-(x+3) = (2x-1)(x+3)$.

3.4 (a) Common factor 2: $2(9x^2-4)$; this is a difference of two squares; $2(3x+2)(3x-2)$.
(b) $a \times c = 18$; two numbers multiplying to 18 and adding to -11 are -9 and -2; $3x^2 - 9x - 2x + 6 = 3x(x-3) - 2(x-3)$; $(3x-2)(x-3)$.
(c) Numerator $= (x+3)(x-3)$; denominator $= (x+4)(x-3)$; cancelling the common $(x-3)$ gives $\dfrac{x+3}{x+4}$.
(d) $\dfrac{x+3}{x+4} = 2$, so $x+3 = 2(x+4)$; $x+3 = 2x+8$, so $-5 = x$; $x = -5$.