

E2.13 Functions

Name: _____ Class: _____ Date: _____

Total: 12 marks

KEY WORDS function 函数 • composite function 复合函数 • inverse function 反函数
• domain 定义域 • function notation 函数记号

Objective

Build the skills to answer Extended exam questions on **functions** 函数 — using **function notation** 函数记号, forming a **composite function** 复合函数, finding an **inverse function** 反函数, and stating a value excluded from the **domain** 定义域.

You must be able to:

- evaluate $f(a)$ and form composite functions such as $fg(x)$ and $gf(x)$
- find an inverse function by rearranging $y = f(x)$ to make x the subject
- state which values of x must be excluded from a domain

1 Worked examples

■ $fg(x)$ means "do g first"

$$f(x) = 3x - 5: \text{ do } \times 3, \text{ then } -5$$



$$f^{-1}(x) = \frac{x+5}{3}: \text{ reverse the order, undo each step}$$



the inverse takes the output back to the input: $f(2) = 1$, so $f^{-1}(1) = 2$

A composite function is two machines in a row — the right-hand letter runs first

- **Evaluate:** if $f(x) = 3x - 1$ then $f(4) = 11$.
- **Composite:** $fg(x) = f(g(x))$ — substitute g INTO f . With $f(x) = 2x$ and $g(x) = x + 3$, $fg(x) = 2(x + 3) = 2x + 6$, while $gf(x) = 2x + 3$. The two are **different**.
- **Inverse:** write $y = f(x)$, rearrange for x , then swap the letters. For $f(x) = 5x + 2$, $y = 5x + 2$ gives $x = \frac{y-2}{5}$, so $f^{-1}(x) = \frac{x-2}{5}$.
- **Domain:** exclude any x that makes a denominator zero or a square root negative.

Answer shapes (marked by **method marks** — M1 for a correct method, A1 for the correct answer): the substitution line, or the rearranged equation before the letters are swapped, is the method mark.

2 Practice

2.1 $f(x) = 5x + 2$. Find $f(3)$. [1]

2.2 $f(x) = 5x + 2$. Find $f^{-1}(x)$. [2]

2.3 $f(x) = 2x$ and $g(x) = x + 3$. Find $fg(x)$ in its simplest form. [2]

3 Exam-style questions

3.1 $f(x) = 3x - 4$ and $g(x) = x^2$.

(a) Find $gf(2)$. [2]

(b) Find $fg(x)$. [2]

(c) Find $f^{-1}(x)$. [2]

3.2 $h(x) = \frac{2}{x-1}$. Write down the value of x that must be excluded from the domain of h . [1]

Solutions

Marking: **M1** a correct method, **A1** the correct answer.

Each answer shows the steps, then the **result**.

2.1

- $5(3) + 2$
 $= 17$

2.2

- $y = 5x + 2$, so $x = \frac{y - 2}{5}$
 $f^{-1}(x) = \frac{x - 2}{5}$

2.3

- do g first, then f : $f(x + 3) = 2(x + 3)$
 $= 2x + 6$

3.1

(a)

- $f(2) = 3(2) - 4 = 2$
- then $g(2) = 2^2$
 $= 4$

(b)

- do g first: $f(x^2) = 3x^2 - 4$
 $= 3x^2 - 4$
- note $gf(x) = (3x - 4)^2$, which is not the same

(c)

- $y = 3x - 4$, so $x = \frac{y + 4}{3}$
 $f^{-1}(x) = \frac{x + 4}{3}$

3.2

- the denominator $x - 1$ must not be zero
 $x = 1$