IGCSE MATHEMATICS • EXERCISE SHEET

E2.12 Differentiation

Name: ______________ Class: __________ Date: ____________ **Total: 17 marks**

KEY WORDS differentiate 求导 • derivative 导数 • turning point 驻点 • maximum 极大值 • minimum 极小值

Objective

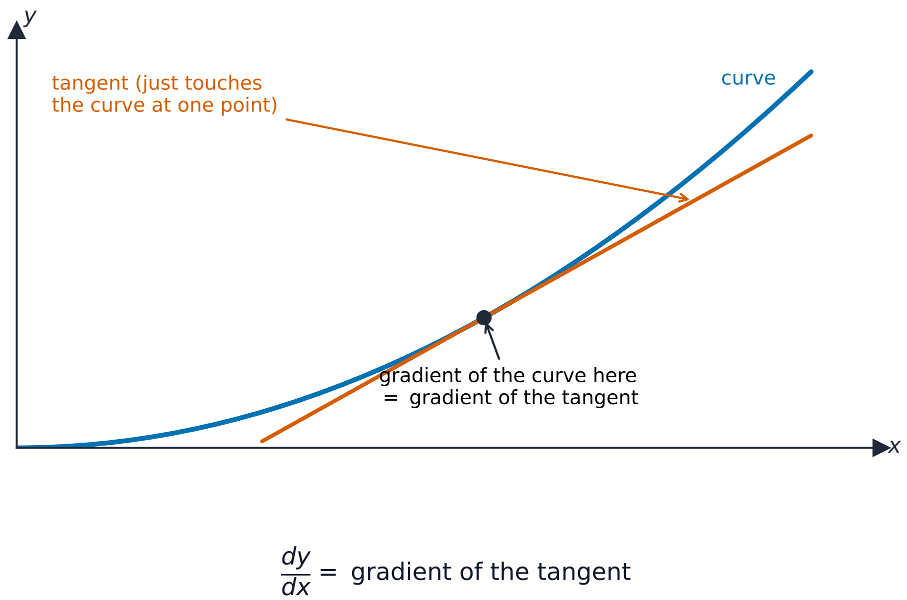
Build the skills to answer Extended exam questions on **differentiation** 微分 — finding the **derivative** 导数 of a polynomial, using it to find a **gradient** 斜率 at a point, and finding **turning points** 驻点 and deciding whether each is a **maximum** 极大值 or a **minimum** 极小值.

You must be able to:

- differentiate a sum of terms of the form ax^n
- find the gradient of a curve at a given point
- find the turning points by solving $\dfrac{dy}{dx} = 0$, and determine their nature

1 Worked examples

■ Multiply by the power, then drop the power by one



The derivative gives the gradient of the tangent at any point

- **The rule:** if $y = ax^n$ then $\dfrac{dy}{dx} = anx^{n-1}$.
- **A constant differentiates to zero**, and ax differentiates to a — its graph is a straight line of gradient a.
- **Gradient at a point:** differentiate, then substitute the x-value. For $y = x^3 - 4x$ at $x = 2$, $\dfrac{dy}{dx} = 3x^2 - 4 = 8$.
- **Turning points:** solve $\dfrac{dy}{dx} = 0$ for x, then put each x back into the ORIGINAL equation for y.
- **Nature:** $\dfrac{d^2y}{dx^2} < 0$ is a maximum, > 0 is a minimum.

Answer shapes (marked by **method marks** — M1 for a correct method, A1 for the correct answer): the derivative itself is a method mark, and setting it to zero is another — so write both lines down.

2 Practice

2.1 Differentiate $y = 3x^4 - 5x^2 + 7$. [3]

2.2 Find the gradient of the curve $y = x^3 - 4x$ at the point where $x = 2$. [3]

2.3 Find the coordinates of the turning point of $y = x^2 - 6x + 5$. [3]

3 Exam-style questions

3.1 A curve has equation $y = 2x^3 - 9x^2 + 12x$.

(a) Find $\dfrac{dy}{dx}$. [2]

(b) Find the coordinates of the two turning points. [4]

(c) Determine, with a reason, which turning point is a maximum and which is a minimum. [2]

Solutions

Marking: **M1** a correct method, **A1** the correct answer.

Each answer shows the steps, then the **result**.

2.1

- $3x^4 \to 12x^3$, $-5x^2 \to -10x$, and the 7 vanishes

$$\frac{dy}{dx} = \mathbf{12x^3 - 10x}$$

2.2

- $\dfrac{dy}{dx} = 3x^2 - 4$
- substitute $x = 2$: $3(4) - 4$

$$= \mathbf{8}$$

2.3

- $\dfrac{dy}{dx} = 2x - 6$, and set it to zero
- $x = 3$
- substitute into the original: $9 - 18 + 5$

$$= (\mathbf{3},\ -\mathbf{4})$$

3.1

(a)

- differentiate term by term

$$\frac{dy}{dx} = \mathbf{6x^2 - 18x + 12}$$

(b)

- set it to zero: $6x^2 - 18x + 12 = 0$, so $x^2 - 3x + 2 = 0$
- $(x-1)(x-2) = 0$, so $x = 1$ or $x = 2$
- $y(1) = 2 - 9 + 12 = 5$ and $y(2) = 16 - 36 + 24 = 4$

$$(\mathbf{1},\ \mathbf{5}) \text{ and } (\mathbf{2},\ \mathbf{4})$$

(c)

- $\dfrac{d^2y}{dx^2} = 12x - 18$
- at $x = 1$ this is -6, which is negative, so $(1,\ 5)$ is a **maximum**; at $x = 2$ it is $+6$, so $(2,\ 4)$ is a **minimum**