

E2.10 Graphs of functions

Name: _____ Class: _____ Date: _____

Total: 28 marks

KEY WORDS table of values 数值表 • coordinates 坐标 • axis 坐标轴 • function 函数 • roots 根

Objective

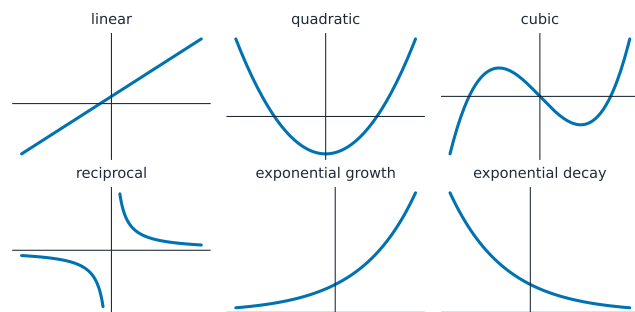
Build the skills to answer Extended exam questions on **graphs of functions** 函数图象 — making a **table of values** 数值表, finding **roots** 根 (where the graph crosses the x -axis), and reading solutions from a graph.

You must be able to:

- complete a table of values for a function
- identify the roots and the y -intercept of a graph
- use a graph to solve an equation

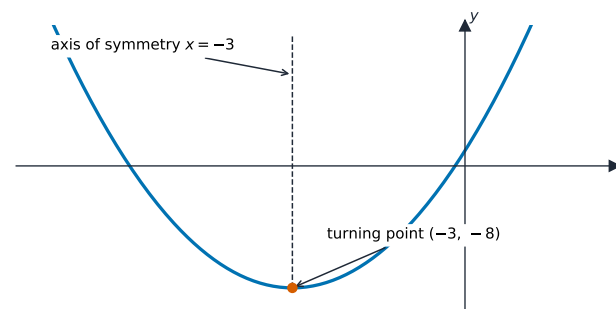
1 Worked examples

■ Tables, roots and intersections



recognise $y = x^2$, $y = 1/x$, $y = 2^x$ from shape

Plot points from a table, then join with a smooth curve



$$y = (x + 3)^2 - 8$$

The quadratic is one key function graph

- **Table:** for $y = x^2 - 3$, when $x = 2$, $y = 4 - 3 = 1$.
- **Roots** 根: where $y = 0$ (the graph crosses the x -axis).
- **Solve from a graph:** the **intersection** 交点 of a line and a curve solves the two together.

Answer shapes: M1 a correct method, A1 the correct answer.

■ Reading a graph, and solving with a line

The **roots** are where the curve crosses the x -axis, so they are the solutions of $y = 0$. The **y -intercept** is the value at $x = 0$. The **turning point** of $y = ax^2 + bx + c$ lies on the line of symmetry $x = -\frac{b}{2a}$; substitute that x back to get its y .

Solving with a drawn line. To solve an equation using a drawn curve, rearrange it so that one side is exactly the curve's equation; the other side is then the line to draw, and the solutions are the x -coordinates where the curve and line cross.

Worked example. The curve $y = x^2 - 2x - 3$ is drawn. Use it to solve $x^2 - 2x - 8 = 0$.

1. Rearrange so the left side matches the drawn curve: $x^2 - 2x - 8 = 0$ becomes $x^2 - 2x - 3 = 5$.
2. So draw the horizontal line $y = 5$.
3. Read the x -coordinates where the curve meets it: $x = -2$ and $x = 4$.
4. Check by factorising: $(x - 4)(x + 2) = 0$, which agrees.

Sketching. Give the shape (a positive x^2 term opens upwards, a negative one opens downwards), the roots, the y -intercept and the turning point. For a cubic, the shape rises from bottom-left to top-right when the x^3 term is positive, and the roots are still where it crosses the x -axis.

2 Practice

2.1 For $y = x^2 - 4$, find the value of y when $x = 3$. [1]

2.2 Write down the y -intercept of $y = 2x^2 - 5$. [1]

2.3 Find the roots of $y = x^2 - 9$. [2]

2.4 Find the coordinates of the turning point of $y = x^2 - 6x + 5$. [3]

2.5 State the line you would draw on the graph of $y = x^2 + x$ to solve $x^2 + x - 4 = 0$. [1]

3 Exam-style questions

3.1 $y = x^2 - 2x - 3$.

(a) Complete the value of y when $x = -2$. [1]

(b) Find the roots of $x^2 - 2x - 3 = 0$. [2]

3.2 The graph of $y = x^2 - 2x - 3$ is drawn. By solving a quadratic, find the x -values where the graph meets the line $y = 5$. [3]

3.3 The curve $y = x^2 - 4x + 1$ is to be drawn for $-1 \leq x \leq 5$.

(a) Complete the table of values.

[3]

x	-1	0	1	2	3	4	5
y							

(b) Write down the coordinates of the turning point of the curve.

[3]

(c) Use the curve to solve $x^2 - 4x + 1 = 0$, giving your answers correct to 1 decimal place.

[2]

(d) By drawing a suitable straight line, explain how you would solve $x^2 - 4x - 2 = 0$, and give the equation of that line.

[2]

(e) Solve $x^2 - 4x - 2 = 0$ algebraically, giving your answers correct to 2 decimal places.

[4]

Solutions

Marking: **M1** a correct method, **A1** the correct answer.

2.1 $y = 3^2 - 4 = 5$.

2.2 the y -intercept is at $x = 0$: $y = -5$.

2.3 $x^2 - 9 = 0 \rightarrow (x + 3)(x - 3) = 0 \rightarrow x = 3$ or $x = -3$.

2.4 $x = -\frac{-6}{2} = 3$; $y = 9 - 18 + 5 = -4$; the turning point is $(3, -4)$.

2.5 $y = 4$.

3.1 (a) $y = (-2)^2 - 2(-2) - 3 = 4 + 4 - 3 = 5$. (b) $(x - 3)(x + 1) = 0 \rightarrow x = 3$ or $x = -1$.

3.2 set $x^2 - 2x - 3 = 5 \rightarrow x^2 - 2x - 8 = 0 \rightarrow (x - 4)(x + 2) = 0 \rightarrow x = 4$ or $x = -2$.

3.3 (a) 6, 1, -2, -3, -2, 1, 6.

(b) $x = -\frac{-4}{2} = 2$; $y = 4 - 8 + 1 = -3$; the turning point is $(2, -3)$.

(c) The curve crosses the x -axis at about $x = 0.3$ and $x = 3.7$.

(d) Rearranging, $x^2 - 4x - 2 = 0$ becomes $x^2 - 4x + 1 = 3$, so draw the line $y = 3$ and read where it meets the curve.

(e) $a = 1$, $b = -4$, $c = -2$, so $x = \frac{4 \pm \sqrt{16 + 8}}{2}$; $\sqrt{24} = 4.899$; $x = 4.45$ or $x = -0.45$.