

E1.7 Indices I

Name: _____ Class: _____ Date: _____ **Total: 22 marks**

KEY WORDS indices 指数 • laws of indices 指数定律 • factor 因数 • power 幂
• prime factor 质因数

Objective

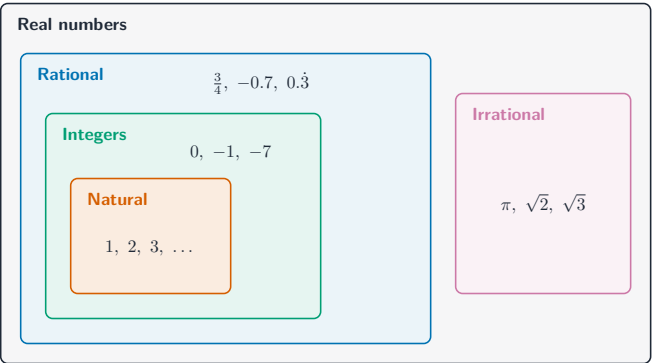
Build the skills to answer Extended exam questions on **indices** 指数 — the **laws of indices** 指数定律, and **zero, negative and fractional** 零、负、分数 powers.

You must be able to:

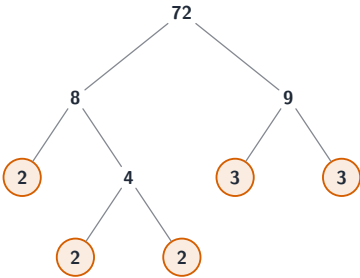
- use $a^m \times a^n = a^{m+n}$, $a^m \div a^n = a^{m-n}$, $(a^m)^n = a^{mn}$
- evaluate a^0 , a^{-n} and $a^{\frac{m}{n}}$
- simplify expressions with indices

1 Worked examples

■ Laws of indices



Add powers to multiply; subtract to divide



$$72 = 2^3 \times 3^2$$

Indices appear when a number is written in prime factors

$$a^m \times a^n = a^{m+n} \quad a^m \div a^n = a^{m-n} \quad (a^m)^n = a^{mn}$$

$$a^0 = 1 \quad a^{-n} = \frac{1}{a^n} \quad a^{\frac{m}{n}} = (\sqrt[n]{a})^m$$

- **Worked:** $2^{-3} \times 2^4 = 2^1 = 2$; $(2^3)^2 = 2^6 = 64$; $8^{-\frac{2}{3}} = \frac{1}{(\sqrt[3]{8})^2} = \frac{1}{4}$.

Answer shapes: M1 a correct method, A1 the correct answer.

2 Practice

2.1 Simplify $x^5 \times x^3$. [1]

2.2 Work out the value of 2^{-3} . [1]

2.3 Work out the value of $81^{\frac{1}{2}}$. [1]

3 Exam-style questions

3.1 Simplify $(2a^2)^3$. [2]

3.2 Work out the value of $27^{\frac{2}{3}}$. [2]

3.3 Work out the value of $16^{-\frac{1}{2}}$. [2]

3.4 (a) Simplify, leaving your answers in index form: [4]

(i) $\frac{x^7 \times x^{-3}}{x^2}$ (ii) $(2y^3)^4$

(b) Evaluate without a calculator, showing each step: [4]

(i) $27^{\frac{2}{3}}$ (ii) $\left(\frac{25}{4}\right)^{-\frac{1}{2}}$

(c) Solve $2^{x+1} = \frac{1}{32}$. [3]

(d) Explain, using the laws of indices, why $a^0 = 1$ for any non-zero a . [2]

Solutions

Marking: **M1** a correct method, **A1** the correct answer.

2.1 $x^5 \times x^3 = x^8$.

2.2 $2^{-3} = \frac{1}{2^3} = \frac{1}{8}$.

2.3 $81^{\frac{1}{2}} = \sqrt{81} = 9$.

3.1 $(2a^2)^3 = 2^3 a^6 = 8a^6$.

3.2 $27^{\frac{2}{3}} = (\sqrt[3]{27})^2 = 3^2 = 9$.

3.3 $16^{-\frac{1}{2}} = \frac{1}{16^{\frac{1}{2}}} = \frac{1}{\sqrt{16}} = \frac{1}{4}$.

3.4 (a) (i) $\frac{x^{7+(-3)}}{x^2} = \frac{x^4}{x^2} = x^2$. (ii) $(2y^3)^4 = 2^4 y^{12} = 16y^{12}$.

(b) (i) $27^{\frac{2}{3}} = (\sqrt[3]{27})^2 = 3^2 = 9$. (ii) $\left(\frac{25}{4}\right)^{-\frac{1}{2}} = \left(\frac{4}{25}\right)^{\frac{1}{2}} = \frac{2}{5}$.

(c) $\frac{1}{32} = 2^{-5}$; so $x + 1 = -5$; $x = -6$.

(d) $\frac{a^n}{a^n} = a^{n-n} = a^0$ by the division law; but any non-zero number divided by itself is 1, so $a^0 = 1$.