IGCSE MATHEMATICS • EXERCISE SHEET

E1.18 Surds

Name: _____ Class: _____ Date: _____ **Total: 14 marks**

KEY WORDS surd 根式 • simplify 化简 • rationalise the denominator 分母有理化 • exact form 精确值 • irrational 无理数

Objective

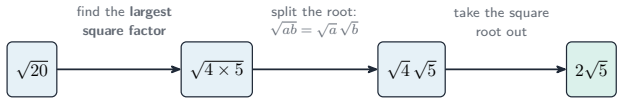
Build the skills to answer Extended exam questions on **surds** 根式 — **simplifying** 化简 a square root, adding and subtracting **like surds** 同类根式, expanding brackets containing surds, and **rationalising the denominator** 分母有理化.

You must be able to:

- simplify a surd by taking out the largest square factor
- add, subtract and multiply surds, leaving the answer in exact form
- rationalise a denominator of the form $\sqrt{a}$

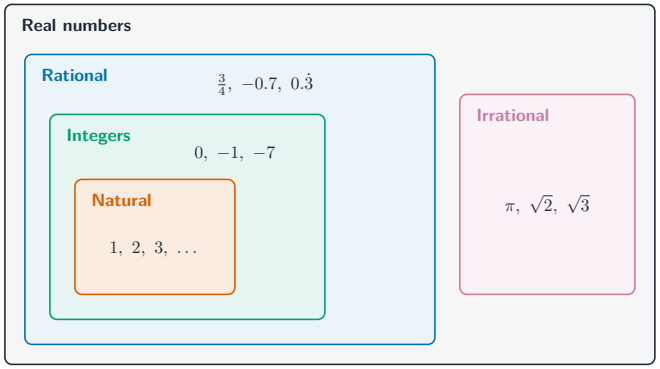
1 Worked examples

■ Pull out the largest square factor, then collect like surds



picking a smaller square factor ($\sqrt{20} = \sqrt{2 \times 10}$) gets stuck - always take out the **largest** square: 4, 9, 16, 25, ...

Look for 4, 9, 16, 25, 36 … inside the root



A surd that does not simplify to a whole number is irrational

- **Simplify:** $\sqrt{50} = \sqrt{25 \times 2} = 5\sqrt{2}$. Take the **largest** square factor, or you will have to simplify twice.
- **Add and subtract:** only **like** surds combine. $\sqrt{12} + \sqrt{27} = 2\sqrt{3} + 3\sqrt{3} = 5\sqrt{3}$.
- **Multiply:** $\left(\sqrt{5}\right)^2 = 5$, and $\left(2 + \sqrt{3}\right)\left(2 - \sqrt{3}\right) = 4 - 3 = 1$ — the surd disappears.
- **Rationalise:** multiply top and bottom by the surd. $\dfrac{6}{\sqrt{3}} = \dfrac{6\sqrt{3}}{3} = 2\sqrt{3}$.

Answer shapes (marked by **method marks** — M1 for a correct method, A1 for the correct answer): the line splitting the root into a square factor times the rest is the method mark. A decimal answer earns nothing — surd questions want **exact** form.

2 Practice

2.1 Simplify $\sqrt{75}$. [2]

2.2 Simplify $\sqrt{8} + \sqrt{18}$. [2]

2.3 Rationalise the denominator of $\frac{10}{\sqrt{5}}$, giving your answer in its simplest form. [2]

3 Exam-style questions

3.1 (a) Expand and simplify $\left(3+\sqrt{2}\right)\left(3-\sqrt{2}\right)$. [2]

(b) Write $\sqrt{48}-\sqrt{12}$ in the form $a\sqrt{3}$, where a is an integer. [2]

3.2 Rationalise the denominator of $\frac{8}{\sqrt{6}}$ and simplify your answer. [2]

3.3 A square has an area of 18 cm^2. Find the exact length of one side, in the form $a\sqrt{b}$ where a and b are integers. [2]

Solutions

Marking: **M1** a correct method, **A1** the correct answer.

Each answer shows the steps, then the **result**.

2.1

- $75 = 25 \times 3$ and 25 is a square

$$\sqrt{75} = \mathbf{5\sqrt{3}}$$

2.2

- $\sqrt{8} = 2\sqrt{2}$ and $\sqrt{18} = 3\sqrt{2}$

$$2\sqrt{2} + 3\sqrt{2} = \mathbf{5\sqrt{2}}$$

2.3

- multiply top and bottom by $\sqrt{5}$: $\dfrac{10\sqrt{5}}{5}$

$$= \mathbf{2\sqrt{5}}$$

3.1

(a)

- expand: $9 - 3\sqrt{2} + 3\sqrt{2} - 2$ — the middle terms cancel

$$= \mathbf{7}$$

(b)

- $\sqrt{48} = 4\sqrt{3}$ and $\sqrt{12} = 2\sqrt{3}$

$$4\sqrt{3} - 2\sqrt{3} = \mathbf{2\sqrt{3}}$$

3.2

- multiply top and bottom by $\sqrt{6}$: $\dfrac{8\sqrt{6}}{6}$
- divide top and bottom by 2

$$= \mathbf{\frac{4\sqrt{6}}{3}}$$

3.3

- side $= \sqrt{18}$, and $18 = 9 \times 2$

$$= \mathbf{3\sqrt{2}} \text{ cm}$$