

E1.17 Exponential growth and decay

Name: _____ Class: _____ Date: _____

Total: 14 marks

KEY WORDS compound interest 复利 • depreciation 折旧 • multiplier 乘数
 • exponential decay 指数衰减 • initial value 初始值

Objective

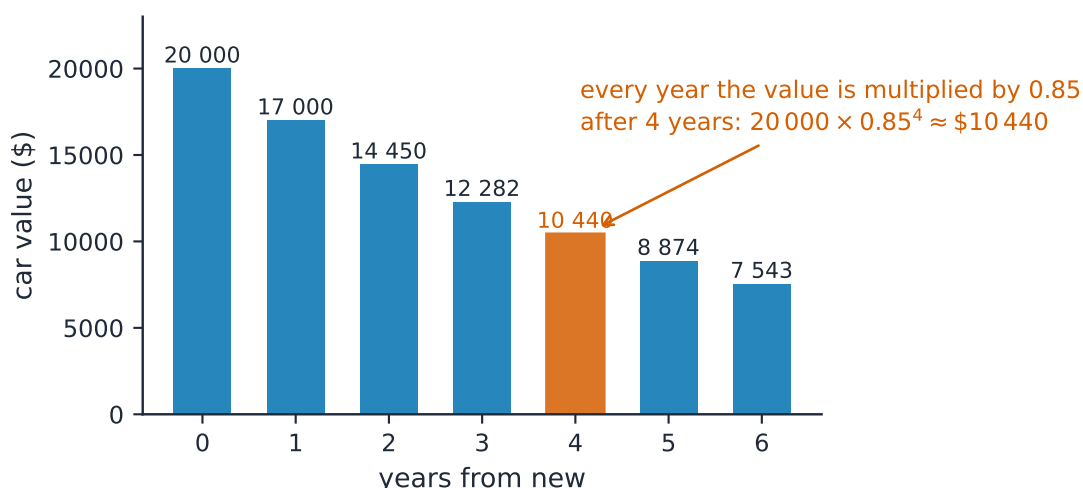
Build the skills to answer Extended exam questions on **exponential growth and decay** 指数增长与衰减 — **compound interest** 复利, **depreciation** 折旧 and **population change** 人口变化, all through a repeated **multiplier** 乘数.

You must be able to:

- write down the multiplier for a given percentage increase or decrease
- use the formula $\text{final} = \text{initial} \times (\text{multiplier})^n$
- work backwards to find the initial amount, or find how many years a change takes

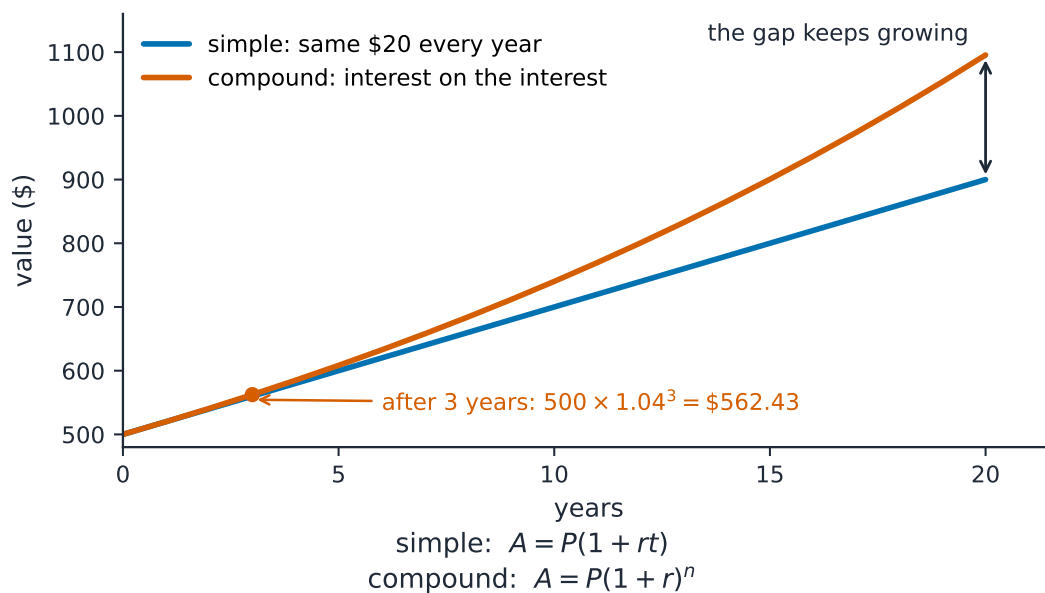
1 Worked examples

■ One multiplier, raised to the number of years



$$V = V_0 r^t \text{ with } 0 < r < 1 \text{ for decay} \cdot \text{ here } r = 0.85 \text{ each year}$$

Each year takes the same fraction, so the fall gets smaller in size but never stops



Compound interest pulls away from simple interest because it earns on interest

- **The multiplier:** an increase of 4% is $\times 1.04$; a decrease of 15% is $\times 0.85$. An increase of 7.5% is $\times 1.075$.
- **Formula:** final = initial $\times$ (multiplier) ^{n} , where n is the number of years.
- **Worked:** \$2000 at 4% compound interest for 3 years = $2000 \times 1.04^3 = \$2249.73$.
- **Backwards:** divide instead of multiplying — initial = final $\div$ (multiplier) ^{n} .

Answer shapes (marked by **method marks** — M1 for a correct method, A1 for the correct answer): writing the multiplier raised to the correct power is the method mark, so show it even if the calculator does the rest.

2 Practice

2.1 Write down the multiplier for an increase of 7.5%.

[1]

2.2 \$5000 is invested at 3% per year compound interest. Find the value after 2 years. [2]

2.3 A population of 8000 falls by 6% each year. Find the population after 3 years, correct to the nearest whole number. [2]

3 Exam-style questions

3.1 A car costs \$24,000 when new and loses 18% of its value each year.

(a) Find its value after 3 years, correct to the nearest dollar. [3]

(b) Find the number of complete years before its value first falls below \$10,000. [3]

3.2 A sum of \$ P is invested at 5% per year compound interest. After 2 years it is worth \$3307.50. Find P . [3]

Solutions

Marking: **M1** a correct method, **A1** the correct answer.

Each answer shows the steps, then the **result**.

2.1

- an increase of 7.5% multiplies by $1 + 0.075$

$$\mathbf{1.075}$$

2.2

- 5000×1.03^2

$$= \mathbf{\$5304.50}$$

2.3

- a fall of 6% gives a multiplier of 0.94, so 8000×0.94^3

$$= 6644.672 = \mathbf{6645}$$

3.1

(a)

- a fall of 18% gives a multiplier of 0.82
- $24\,000 \times 0.82^3$

$$= 13\,232.83 \dots = \mathbf{\$13\,233}$$

(b)

- try successive years: $24\,000 \times 0.82^4 = \$10\,850.92$, still above \$10,000
- $24\,000 \times 0.82^5 = \8897.76 , which is below \$10,000

5 years

3.2

- the multiplier is 1.05, so $P \times 1.05^2 = 3307.50$
- $P = 3307.50 \div 1.1025$

$$= \mathbf{\$3000}$$