

C9.3 Averages and range

Name: _____ Class: _____ Date: _____

Total: 20 marks

KEY WORDS median 中位数 • mean 平均数 • range 极差 • mode 众数 • frequency 频数

Objective

Use averages and range to solve Core IGCSE Mathematics problems.

You must be able to:

- Find mean, median, mode and range from lists or frequencies
- Use frequency-weighted totals for a table mean
- Calculate a missing value from a given mean

1 Key knowledge and worked examples

■ Key knowledge: Averages and range

Mean is sum divided by count. Median is the middle value after sorting; with an even count, average the two middle values. Mode is the most frequent value; there may be more than one or none. Range is maximum minus minimum. For an ungrouped frequency table, $\bar{x} = \sum fx / \sum f$ and cumulative counts locate the median. An extreme value can change mean and range a lot; median is less affected.

■ Worked example: Averages and range

sorted: 2,4,4,7,8	middle: 4	sum: 25
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For 2, 4, 4, 7, 8, $\bar{x} = (2 + 4 + 4 + 7 + 8)/5 = 5$, median 4, mode 4, range $R = 8 - 2 = 6$.

- For 2, 5, 7, 10, median $M = (5 + 7)/2 = 6$.
- A frequency table $x = 1, 2, 3$ with $f = 2, 3, 1$ has $\sum f = 6$, $\sum fx = 2 + 6 + 3 = 11$, so mean $11/6$.
- Values in positions 3 and 4 are both 2, so its median is 2.

■ Method check

A frequency-table mean divides the total of all observations by the total frequency. Do not divide by the number of different values.

2 Practice

2.1 Find the mean of 3, 5, 7, 9. [2]

2.2 Find the median of 8, 1, 6, 3, 5. [2]

2.3 Find the mode and range of 2, 6, 6, 8, 11. [2]

2.4 Values 0, 1, 2 have frequencies 2, 4, 2. Find the mean. [2]

3 Exam-style questions

3.1 Apply the averages and range methods below. Show working and include units where appropriate.

(a) The scores are 4, 7, 7, 8, 9, 13. Find their mean and median. [2]

(b) Find their mode and range. [2]

(c) The score 13 is corrected to 25. Find the new mean and median, and state which changes less. [2]

3.2 A measurement table has masses 1, 2, 3, 4 kg with frequencies 2, 3, 4, 1 respectively.

(a) Find the mean mass, median mass and mode. [3]

(b) An eleventh mass is added and the new mean is exactly 2.5 kg. Find the added mass and the new range. [3]

Solutions

Award method and answer marks as indicated; accept equivalent correct methods.

2.1

- $\bar{x} = 24/4 = \mathbf{6}$.

2.2

- Sort to 1, 3, 5, 6, 8; the middle is **5**.

2.3

- Most frequent gives **mode 6**; $R = 11 - 2 = \mathbf{9}$.

2.4

- $\bar{x} = (0(2) + 1(4) + 2(2))/(2 + 4 + 2) = 8/8 = \mathbf{1}$.

3.1

(a)

- $\bar{x} = 48/6 = \mathbf{8}$; median $M = (7 + 8)/2 = \mathbf{7.5}$.

(b)

- **Mode 7**; $R = 13 - 4 = \mathbf{9}$.

(c)

- New sum is 60: $\bar{x} = 60/6 = \mathbf{10}$; median stays **7.5**, so **median changes less**.

3.2

(a)

- Weighted total $T = 1(2) + 2(3) + 3(4) + 4(1) = 24$; total frequency 10, so mean **2.4 kg**.
- Positions 5 and 6 are 2 and 3, so median **2.5 kg**.
- Highest frequency gives mode **3 kg**.

(b)

- New total $T_n = 11(2.5) = 27.5$.
- Added mass $x = T_n - 24 = \mathbf{3.5}$ kg.
- Extremes stay 1 and 4, so new range $R = 4 - 1 = \mathbf{3}$ kg.