

C8.3 Probability of combined events

Name: _____ Class: _____ Date: _____ **Total: 20 marks**

KEY WORDS with replacement 有放回 • combined events 组合事件 • venn diagram 维恩图
• probability 概率 • fair 公平

Objective

Use probability of combined events to solve Core IGCSE Mathematics problems.

You must be able to:

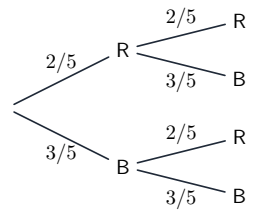
- Use sample spaces and probability trees for independent stages
- Multiply along paths and add exclusive paths
- Distinguish exactly one from at least one outcome

1 Key knowledge and worked examples

■ Key knowledge: Probability of combined events

For two independent stages, multiply along a tree path and add probabilities of different paths for the required event. With replacement, the second draw has the same probabilities as the first. Label probabilities beside branches and outcomes at their ends. A sample-space table lists every ordered pair once. In a two-set Venn diagram, add disjoint regions and count overlap once.

■ Worked example: Probability of combined events



A bag has 2 red and 3 blue counters. Draw twice, replacing the first counter.

- Each stage has red $\frac{2}{5}$, blue $\frac{3}{5}$.
- Both red: $p = (\frac{2}{5})(\frac{2}{5}) = \frac{4}{25}$.
- Exactly one red: paths RB and BR, so $p = (\frac{2}{5})(\frac{3}{5}) + (\frac{3}{5})(\frac{2}{5}) = \frac{12}{25}$.
- A coin and a die give 12 ordered outcomes; head and an even die number covers 3, so $p = \frac{3}{12} = \frac{1}{4}$.

■ Method check

Exactly one success uses the two mixed paths. At least one success also includes two successes; its complement is no successes.

2 Practice

2.1 Two fair coins are tossed. List their ordered sample space. [2]

2.2 For two fair coins, find the probability of exactly one head. [2]

2.3 A bag has 3 green and 1 white counter. Two draws are made with replacement. Find the probability of two whites. [2]

2.4 A fair coin and a fair die are used. Find the probability of a tail and a number above 4. [2]

3 Exam-style questions

3.1 Apply the probability of combined events methods below. Show working and include units where appropriate.

(a) A bag has 1 red and 2 blue counters. Draw twice with replacement. Draw and label a probability tree. [2]

(b) Find the probability of exactly one red. [2]

(c) Find the probability of at least one red, and check using the complement. [2]

3.2 A spinner has 3 equally sized red sectors and 1 blue sector. It is spun twice independently.

(a) Find the probability of exactly one blue, showing both paths. [3]

(b) Find the probability of at least one blue using a complement. Explain why it exceeds the answer to part (a). [3]

Solutions

Award method and answer marks as indicated; accept equivalent correct methods.

2.1

- Each sequence occurs once: **HH, HT, TH, TT**.

2.2

- Two of four outcomes qualify: $p = 2/4 = \mathbf{1/2}$.

2.3

- White stays 1/4: $p = (1/4)^2 = \mathbf{1/16}$.

2.4

- $p = (1/2)(2/6) = \mathbf{1/6}$, using independence.

3.1

(a)

- First branches R 1/3, B 2/3; each splits again into R 1/3, B 2/3.** End labels are RR, RB, BR, BB.

(b)

- $p = (1/3)(2/3) + (2/3)(1/3) = \mathbf{4/9}$.

(c)

- $p = 1 - (2/3)^2 = \mathbf{5/9}$; also $1/9 + 4/9 = 5/9$.

3.2

(a)

- Each stage has $P(B) = 1/4$ and $P(R) = 3/4$.
- Add BR and RB: $p = (1/4)(3/4) + (3/4)(1/4)$, giving $p = \mathbf{3/8}$.

(b)

- Complement is two reds: $q = 1 - (3/4)^2$, so $q = \mathbf{7/16}$.
- It also includes BB, probability 1/16, which exactly-one excludes.