

C8.2 Relative and expected frequencies

Name: _____ Class: _____ Date: _____

Total: 20 marks

KEY WORDS random 随机 • bias 偏倚 • fair 公平 • expected frequency 期望频数
• relative frequency 相对频率

Objective

Use relative and expected frequencies to solve Core IGCSE Mathematics problems.

You must be able to:

- Estimate probability from relative frequency
- Use an estimated probability to predict an expected count
- Explain sampling variation and limitations of predictions

1 Key knowledge and worked examples

■ Key knowledge: Relative and expected frequencies

Relative frequency is observed successes divided by trials; it estimates probability. Expected frequency is probability multiplied by number of trials. Random selection gives each equally eligible member a fair chance. A fair device has the stated equally likely outcomes; bias changes those probabilities. Larger experiments usually give more stable estimates, but neither an estimate nor an expected count guarantees the next result.

■ Worked example: Relative and expected frequencies

36 red	120 trials	relative frequency
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A spinner lands on red 36 times in 120 spins.

- Relative frequency: $p \approx 36/120 = 0.3$.
- Expected reds in 500 further spins: $E = np = 500(0.3) = 150$.
- This is an estimate, not a promise of exactly 150. One unusual result alone does not prove bias.

■ Method check

An expected count is a prediction over repeated trials, not a guarantee. Combine counts before calculating a relative frequency from unequal sample sizes.

2 Practice

2.1 A coin shows heads 44 times in 80 tosses. Estimate the probability of heads. [2]

2.2 An event has probability 0.2. Find its expected frequency in 150 trials. [2]

2.3 A fair die is rolled 300 times. Find the expected number of sixes. [2]

2.4 Explain why 20 heads in 30 tosses does not prove a coin is biased. [2]

3 Exam-style questions

3.1 Apply the relative and expected frequencies methods below. Show working and include units where appropriate.

- (a) A factory checks 240 items and finds 18 faulty. Estimate the fault probability. [2]

- (b) Use the estimate to find the expected faulty count in 800 items. [2]

- (c) Explain one limitation of this prediction. [2]

3.2 Machine A produces 8 faulty items in a sample of 100. Machine B produces 21 faulty items in a sample of 300.

- (a) Estimate each fault probability and state which machine has the lower estimated rate. [3]

- (b) Pool both samples to estimate the fault probability. Predict the expected number faulty in 1200 items from the same mixture and state one limitation. [3]

Solutions

Award method and answer marks as indicated; accept equivalent correct methods.

2.1

- $p \approx 44/80 = \mathbf{0.55}$.

2.2

- $E = np = 150(0.2) = \mathbf{30}$.

2.3

- $E = np = 300(1/6) = \mathbf{50}$.

2.4

- **Random variation can give uneven counts in a small sample; more trials are needed to judge the long-run pattern.**

3.1

(a)

- $p \approx 18/240 = \mathbf{0.075}$.

(b)

- $E = np = 800(0.075) = \mathbf{60}$.

(c)

- **The rate is an estimate and can change; the actual count may differ even if production stays similar.**

3.2

(a)

- $p_A = 8/100 = \mathbf{0.08}$ and $p_B = 21/300 = \mathbf{0.07}$.
- B has the lower estimated rate, although the samples do not prove its true rate is lower.

(b)

- Pooled estimate $p = (8 + 21)/(100 + 300) = \mathbf{29/400}$.
- Expected count $E = np = 1200(29/400) = \mathbf{87}$.
- Accept sampling variation or a changed production mixture/rate as a limitation.