

C7.1 Transformations

Name: _____ Class: _____ Date: _____

Total: 20 marks

KEY WORDS reflection 反射 • translation 平移 • rotation 旋转 • centre 中心
• scale factor 比例因子

Objective

Use transformations to solve Core IGCSE Mathematics problems.

You must be able to:

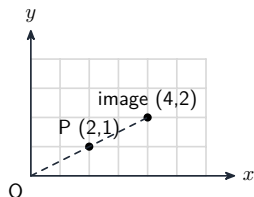
- Perform single translations, reflections, rotations and enlargements
- State all parameters needed to describe a transformation
- Use a nonorigin centre and positive fractional enlargement factor

1 Key knowledge and worked examples

■ Key knowledge: Transformations

A translation moves every point by the same horizontal and vertical amounts, written as a column vector. A reflection uses a mirror line; corresponding points are perpendicular to it and equally far away. A rotation needs angle, direction and centre. An enlargement needs centre and scale factor: multiply each distance from the centre. Core uses horizontal or vertical mirrors, multiples of 90° and positive scale factors, including fractions; no combined transformations.

■ Worked example: Transformations



For $P = (2, 1)$:

- Translation 3 right and 2 down gives $P' = (2 + 3, 1 - 2) = (5, -1)$.
- Reflection in $x = 0$ gives $P' = (-2, 1)$; in $y = 2$ gives $P' = (2, 3)$.
- Rotation 90° anticlockwise about the origin changes (x, y) to $(-y, x)$, giving $(-1, 2)$.

- Enlargement centred at origin with factor $1/2$ gives $(1, 0.5)$. About $(1, 1)$ with factor 2: displacement $(1, 0)$ doubles to $(2, 0)$, giving $(3, 1)$.

■ Method check

Treat each requested transformation separately unless told otherwise. A rotation needs centre, angle and direction; an enlargement needs centre and scale factor.

2 Practice

2.1 Translate $(1, -3)$ by $\begin{pmatrix} 4 \\ 2 \end{pmatrix}$. [2]

2.2 Reflect $(3, 5)$ in the line $y = 2$. [2]

2.3 Rotate $(4, 1)$ by 90° clockwise about the origin. [2]

2.4 Enlarge $(6, 4)$ by scale factor $1/2$, centred at the origin. [2]

3 Exam-style questions

3.1 Apply the transformations methods below. Show working and include units where appropriate.

(a) Triangle A has vertices (1, 1), (3, 1), (1, 2). Draw its image after enlargement by factor 2 about the origin. [2]

(b) A separate image of A has vertices (4, 0), (6, 0), (4, 1). Describe this single transformation completely. [2]

(c) A separate image has vertices (−1, −1), (−3, −1), (−1, −2). Describe this single rotation completely. [2]

3.2 Triangle A has vertices (2,2), (6,2), (2,6). Each transformation below acts on the original triangle.

(a) Find the image vertices under enlargement with centre (2,2) and scale factor one half. [3]

(b) Find the image vertices under reflection in $x = 4$. Explain how perpendicular distances from the mirror check the reflected points. [3]

Solutions

Award method and answer marks as indicated; accept equivalent correct methods.

2.1

- Add the vector components: $(5, -1)$.

2.2

- The point is 3 above the line, so its image is 3 below: $(3, -1)$.

2.3

- Clockwise changes (x, y) to $(y, -x)$: $(1, -4)$.

2.4

- Multiply both coordinates: $(3, 2)$.

3.1

(a)

- Double each coordinate: $(2, 2)$, $(6, 2)$, $(2, 4)$; join with a ruler.

(b)

- Every point moves 3 right, 1 down: **translation by** $\begin{pmatrix} 3 \\ -1 \end{pmatrix}$.

(c)

- **Rotation** 180° **about the origin**; either direction is valid for a half-turn.

3.2

(a)

- Keep the centre **(2,2)** fixed.
- Halve each displacement from it, giving the other image vertices **(4,2)** and **(2,4)**.

(b)

- Reflection gives **(6,2)**, **(2,2)**, **(6,6)**. The x-values 2 and 6 are both **2 units from x = 4** on opposite sides.
- Each reflected point keeps its y-coordinate, so the joining segment is perpendicular to the vertical mirror.