

# C5.4 Surface area and volume

Name: \_\_\_\_\_ Class: \_\_\_\_\_ Date: \_\_\_\_\_

Total: 20 marks

**KEY WORDS** cylinder 圆柱 • cuboid 长方体 • surface area 表面积 • sphere 球  
• cross-section 横截面

## Objective

Use surface area and volume to solve Core IGCSE Mathematics problems.

**You must be able to:**

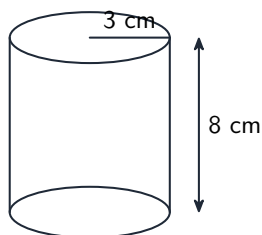
- Find prism, cylinder, cone, pyramid and sphere volumes
- Calculate surface area from the exposed faces
- Use perpendicular height for volume and slant height where area requires it

## 1 Key knowledge and worked examples

### ■ Key knowledge: Surface area and volume

Volume measures space inside; surface area totals exposed faces. Cuboid:  $V = lwh$ ,  $S = 2(lw + lh + wh)$ . Prism:  $V = Ah$ , where  $A$  is cross-section area and  $h$  is length. Cylinder:  $V = \pi r^2 h$ , curved area  $2\pi rh$ ; add circular ends when needed. Cone:  $V = \pi r^2 h/3$ , curved area  $\pi rl$ , using slant height  $l$ . Pyramid:  $V = Ah/3$ . Sphere:  $V = 4\pi r^3/3$ ,  $S = 4\pi r^2$ .

### ■ Worked example: Surface area and volume



A closed cylinder has radius 3 cm and height 8 cm.

- Volume:  $V = \pi r^2 h = \pi(3)^2(8) = 72\pi \text{ cm}^3$ .
- Surface area:  $S = 2\pi rh + 2\pi r^2 = 2\pi(3)(8) + 2\pi(3)^2 = 66\pi \text{ cm}^2$ .
- A cone with  $r = 3$ ,  $h = 4$ ,  $l = 5$  has  $V = \pi(3)^2(4)/3 = 12\pi$  and total  $S = \pi(3)(5) + \pi(3)^2 = 24\pi$  in the appropriate units.
- A triangular prism with cross-section base 6 cm, height 4 cm and length 9 cm has  $A = bh/2 = 6(4)/2 = 12 \text{ cm}^2$ , then  $V = Ah = 12(9) = 108 \text{ cm}^3$ .

- For a pyramid of base area  $30 \text{ cm}^2$  and height  $8 \text{ cm}$ ,  $V = Ah/3 = 30(8)/3 = 80 \text{ cm}^3$ . A sphere of radius  $3 \text{ cm}$  has  $V = 4\pi r^3/3 = 36\pi \text{ cm}^3$  and  $S = 4\pi r^2 = 36\pi \text{ cm}^2$ .

### ■ Method check

Draw or list the exposed faces before adding areas. A prism's volume uses its cross-section area multiplied by the distance between its ends.

## 2 Practice

**2.1** Find the volume and surface area of a  $5 \times 3 \times 2 \text{ cm}$  cuboid. [2]

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**2.2** A triangular prism has cross-section base  $6 \text{ cm}$  and height  $4 \text{ cm}$ ; length is  $9 \text{ cm}$ . Find its volume. [2]

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**2.3** A pyramid has base area  $30 \text{ cm}^2$  and perpendicular height  $8 \text{ cm}$ . Find its volume. [2]

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**2.4** Find the volume and surface area of a sphere of radius  $3 \text{ cm}$  in terms of  $\pi$ . [2]

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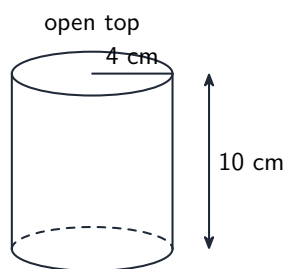


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### 3 Exam-style questions

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**3.1** Apply the surface area and volume methods below. Show working and include units where appropriate.



(a) An open-top cylindrical container has radius 4 cm and height 10 cm. Find its volume in terms of  $\pi$ . [2]

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(b) Find its exposed area, including its circular base but not its open top. [2]

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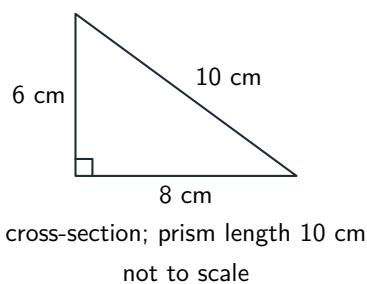
(c) A cone has the same radius and height. Find its volume, and compare it with the cylinder. [2]

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**3.2** A triangular prism is 10 cm long. Its right-triangle cross-section has perpendicular sides 6 cm and 8 cm and hypotenuse 10 cm.



(a) Find the prism volume. [3]

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(b) Find its total surface area, explaining how the two ends and three rectangular faces are included. [3]

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## Solutions

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Award method and answer marks as indicated; accept equivalent correct methods.

### 2.1

- $V = 5(3)(2) = \mathbf{30 \text{ cm}^3}$ ;  $S = 2(15 + 10 + 6) = \mathbf{62 \text{ cm}^2}$ .

### 2.2

- $A = 6(4)/2 = 12$ ;  $V = Ah = 12(9) = \mathbf{108 \text{ cm}^3}$ .

### 2.3

- $V = Ah/3 = 30(8)/3 = \mathbf{80 \text{ cm}^3}$ .

### 2.4

- $V = 4\pi(3)^3/3 = \mathbf{36\pi \text{ cm}^3}$ ;  $S = 4\pi(3)^2 = \mathbf{36\pi \text{ cm}^2}$ .

### 3.1

(a)

- $V = \pi r^2 h = \pi(4)^2(10) = \mathbf{160\pi \text{ cm}^3}$ .

(b)

- $S = 2\pi r h + \pi r^2 = 80\pi + 16\pi = \mathbf{96\pi \text{ cm}^2}$ .

(c)

- $V_{\text{cone}} = \pi r^2 h/3 = \mathbf{160\pi/3 \text{ cm}^3}$ , **one third** of the cylinder volume.

### 3.2

(a)

- Use  $A = bh/2$  for the triangular cross-section.
- $A = 6(8)/2 = 24 \text{ cm}^2$ .
- Then  $V = Al = 24(10) = \mathbf{240 \text{ cm}^3}$ .

(b)

- Two ends give  $S_e = 2(24) = 48 \text{ cm}^2$ .
- Rectangular faces give  $S_r = 10(6 + 8 + 10) = 240 \text{ cm}^2$ .
- Total  $S = S_e + S_r = \mathbf{288 \text{ cm}^2}$ .