

C5.3 Circles, arcs and sectors

Name: _____ Class: _____ Date: _____ **Total: 20 marks**

KEY WORDS circle 圆 • radius 半径 • circumference 圓周 • diameter 直径 • sector 扇形

Objective

Use circles, arcs and sectors to solve Core IGCSE Mathematics problems.

You must be able to:

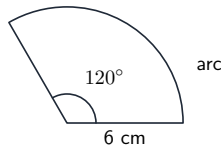
- Use radius and diameter correctly in circle formulas
- Use the sector fraction for arcs and areas
- Include straight radii in a sector perimeter

1 Key knowledge and worked examples

■ **Key knowledge: Circles, arcs and sectors**

For a circle, circumference $C = 2\pi r = \pi d$ and area $A = \pi r^2$. A sector angle θ gives fraction $f = \theta/360$ of the circle. Arc length is $L = f(2\pi r)$; sector area is $S = f\pi r^2$. A sector perimeter includes its two radii: $P = L + 2r$. Core sector angles here are factors of 360° . Keep π exact when asked.

■ **Worked example: Circles, arcs and sectors**



For radius 6 cm and sector angle 120° , the fraction is $f = 120/360 = 1/3$.

- Arc: $L = f(2\pi r) = (1/3)(12\pi) = 4\pi$ cm.
- Sector area: $S = f\pi r^2 = (1/3)(36\pi) = 12\pi$ cm².
- Sector perimeter: $P = L + 2r = 4\pi + 12$ cm.

■ **Method check**

The arc is only the curved part of a sector boundary. Add the two radii for the whole perimeter, and keep pi exact until a decimal answer is requested.

2 Practice

2.1 Find the circumference of a circle of diameter 10 cm, in terms of π . [2]

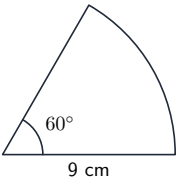
2.2 Find the area of a circle of radius 4 cm, in terms of π . [2]

2.3 Find the arc length for a 90° sector of radius 8 cm. [2]

2.4 Find the area of a semicircle of radius 5 cm. [2]

3 Exam-style questions

3.1 Apply the circles, arcs and sectors methods below. Show working and include units where appropriate.

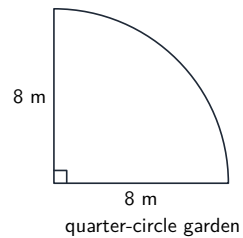


(a) A sector has radius 9 cm and angle 60° . Find its arc length in terms of π . [2]

(b) Find its area in terms of π . [2]

(c) Find its perimeter to one decimal place. [2]

3.2 A quarter-circle garden has radius 8 m.



(a) Find the garden area and the curved boundary length in terms of pi. [3]

(b) A fence follows the whole boundary except for a 2 m gate in one straight edge. Find the fencing length to one decimal place and explain which edges you count. [3]

Solutions

Award method and answer marks as indicated; accept equivalent correct methods.

2.1

- $C = \pi d = \mathbf{10\pi}$ cm.

2.2

- $A = \pi r^2 = \pi(4)^2 = \mathbf{16\pi}$ cm².

2.3

- $L = (90/360)(2\pi \times 8) = \mathbf{4\pi}$ cm.

2.4

- $S = (1/2)\pi(5)^2 = \mathbf{12.5\pi}$ cm².

3.1

(a)

- $L = (60/360)(2\pi \times 9) = \mathbf{3\pi}$ cm.

(b)

- $S = (60/360)\pi(9)^2 = \mathbf{13.5\pi}$ cm².

(c)

- $P = L + 2r = 3\pi + 18 = 27.4247 \dots = \mathbf{27.4}$ cm.

3.2

(a)

- Fraction $f = 90/360 = 1/4$.
- Area $A = f\pi r^2 = \mathbf{16\pi}$ m².
- Arc $L = f(2\pi r) = \mathbf{4\pi}$ m.

(b)

- Count the arc and both radii, then subtract the gate.
- $F = L + 2r - 2 = 4\pi + 16 - 2$, so $F \approx \mathbf{26.6}$ m.