

C4.7 Circle theorems

Name: _____ Class: _____ Date: _____ **Total: 20 marks**

KEY WORDS circle 圆 • tangent 切线 • circumference 圆周 • triangle 三角形 • radius 半径

Objective

Use circle theorems to solve Core IGCSE Mathematics problems.

You must be able to:

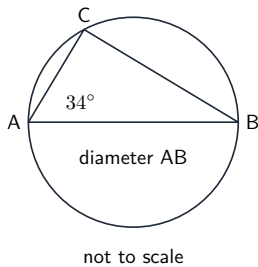
- Identify a diameter and an angle in a semicircle
- Use the perpendicular tangent-radius property
- Combine a circle fact with triangle-angle sums and give reasons

1 Key knowledge and worked examples

■ **Key knowledge: Circle theorems**

Core circle theorems use two facts: an angle at the circumference standing on a diameter is 90° , and a tangent is perpendicular to the radius at the point of contact. Confirm that the line is actually a diameter or tangent before applying a theorem. Then combine with triangle or straight-line angle sums, stating each reason.

■ **Worked example: Circle theorems**



AB is a diameter and C is on the circle. If angle CAB is 34° :

- Angle ACB is 90° , because it is in a semicircle.
- Triangle sum gives $B = 180 - 90 - 34 = 56^\circ$.
- A tangent at T meets radius OT at 90° . In triangle OTP with angle TOP 42° , angle OPT is $180 - 90 - 42 = 48^\circ$.

■ **Method check**

The 90-degree semicircle fact requires a diameter, not just any chord. At a tangent, the right angle is with the radius drawn to the contact point.

2 Practice

2.1 AB is a diameter and C is a different point on the circle. Find angle ACB and give a reason. [2]

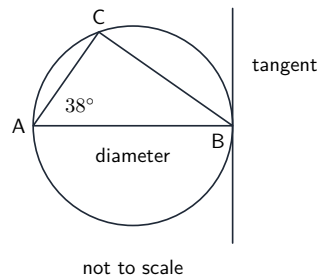
2.2 In that triangle, angle ABC is 61° . Find angle CAB. [2]

2.3 OT is a radius and PT a tangent at T. Find angle OTP with a reason. [2]

2.4 In triangle OTP, angle OPT is 27° . Find angle TOP. [2]

3 Exam-style questions

3.1 Apply the circle theorems methods below. Show working and include units where appropriate.

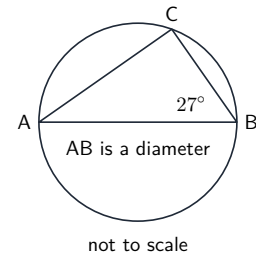


(a) A circle has diameter AB; C lies on its circumference. Angle CAB is 38° . Find angle ACB with a reason. [2]

(b) Find angle ABC with a reason. [2]

(c) The tangent at B meets AB extended. Find its angle with BA, giving a reason. [2]

3.2 AB is a diameter of a circle. C is on the circumference and angle ABC = 27° .



(a) Find angles ACB and CAB, giving the circle fact and triangle fact used. [3]

(b) In a separate circle with centre O, PT is tangent at T. Angle OPT = 32° . Find angle OTP and angle POT, giving reasons. [3]

Solutions

Award method and answer marks as indicated; accept equivalent correct methods.

2.1

- 90° , angle in a semicircle.

2.2

- $A = 180 - 90 - 61 = \mathbf{29^\circ}$, using the triangle sum.

2.3

- 90° , tangent perpendicular to radius at contact.

2.4

- $O = 180 - 90 - 27 = \mathbf{63^\circ}$, using the triangle sum.

3.1

(a)

- 90° , angle in a semicircle.

(b)

- $B = 180 - 90 - 38 = \mathbf{52^\circ}$, triangle angle sum.

(c)

- 90° ; BA lies along the radius at B, and tangent is perpendicular to radius.

3.2

(a)

- $\angle ACB = \mathbf{90^\circ}$ because an angle in a semicircle is a right angle.
- Triangle angles sum to 180° , so $\angle CAB = 180 - 90 - 27 = \mathbf{63^\circ}$.

(b)

- $\angle OTP = \mathbf{90^\circ}$ because a tangent is perpendicular to the radius at contact.
- Triangle sum gives $\angle POT = 180 - 90 - 32$, so $\angle POT = \mathbf{58^\circ}$.