

C4.5 Symmetry

Name: _____ Class: _____ Date: _____

Total: 20 marks

KEY WORDS rotational symmetry 旋转对称 • line symmetry 轴对称 • triangle 三角形
 • equilateral 等边 • square 正方形

Objective

Use symmetry to solve Core IGCSE Mathematics problems.

You must be able to:

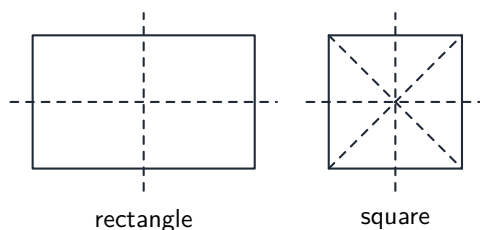
- Identify lines of symmetry of plane figures
- Find rotational order and the smallest matching turn
- Distinguish reflection symmetry from rotation symmetry

1 Key knowledge and worked examples

■ Key knowledge: Symmetry

A line of symmetry reflects a shape onto itself. Rotational symmetry order is the number of matches in one full turn, including the starting position at 360° . The smallest positive matching turn is $360^\circ/n$ for order n . A regular polygon with n sides has n symmetry lines and rotation order n . Order 1 means no smaller matching turn.

■ Worked example: Symmetry



A nonsquare rectangle has 2 lines of symmetry, through the midpoints of opposite sides, and rotation order 2, because a 180° turn matches.

- A square has 4 lines and order 4; the smallest turn is $360/4 = 90^\circ$.
- An equilateral triangle has 3 lines and order 3.
- A scalene triangle has no line symmetry and order 1. An ordinary parallelogram has no lines but order 2.

■ Method check

Rotational symmetry does not imply a mirror line. A half-turn can match an ordinary parallelogram even though reflecting it does not.

2 Practice

2.1 Give the line-symmetry count and rotation order of a square. [2]

2.2 Give the line-symmetry count and rotation order of a non-equilateral isosceles triangle. [2]

2.3 A shape has rotation order 6. Find its smallest matching turn. [2]

2.4 Give both symmetry counts for a regular pentagon. [2]

3 Exam-style questions

3.1 Apply the symmetry methods below. Show working and include units where appropriate.

- (a) A logo is a regular octagon. State its number of lines of symmetry. [2]

- (b) Find its rotational symmetry order and smallest matching turn. [2]

- (c) A second logo is a nonsquare rectangle. Compare its two symmetry counts with the octagon. [2]

3.2 Logo A is a regular pentagon. Logo B is a parallelogram that is neither a rectangle nor a rhombus.

- (a) State A's number of symmetry lines, rotation order and smallest positive matching turn. [3]

- (b) State B's number of symmetry lines and rotation order. Explain why the absence of a mirror line does not make its rotation order 1. [3]

Solutions

Award method and answer marks as indicated; accept equivalent correct methods.

2.1

- Both diagonals and both midlines give **4 lines**; quarter-turn matches give **order 4**.

2.2

- Only the apex-to-base-midpoint line works: **1 line**; **order 1**.

2.3

- $\theta = 360/6 = \mathbf{60^\circ}$.

2.4

- Five vertices give **5 lines**; **order 5**.

3.1

(a)

- 8 lines**, through opposite vertices or opposite side midpoints.

(b)

- Order 8**; $\theta = 360/8 = \mathbf{45^\circ}$.

(c)

- The rectangle has **2 lines and order 2**, fewer than the octagon in both cases.

3.2

(a)

- A has **5 lines**, rotation order **5**, and smallest turn $\theta = 360/5 = \mathbf{72^\circ}$.

(b)

- B has **0 symmetry lines** and rotation order **2**.
- A 180° turn swaps opposite vertices and maps the figure onto itself even though no reflection does.