

C2.7 Sequences

Name: _____ Class: _____ Date: _____

Total: 20 marks

KEY WORDS sequence 数列 • term-to-term rule 递推规则 • cubic 三次 • term 项
 • constant 常数

Objective

Use sequences to solve Core IGCSE Mathematics problems.

You must be able to:

- Continue sequences and identify their term-to-term rule
- Find linear, simple quadratic and simple cubic n th terms
- Check whether a proposed term has an integer index

1 Key knowledge and worked examples

■ Key knowledge: Sequences

A term-to-term rule tells how to move to the next term; an n th-term rule finds any term directly. For a linear sequence with first term a and difference d , $u_n = a + (n-1)d$. For simple quadratic or cubic sequences, compare with n^2 or n^3 and look for a constant addition or multiplication. Check a proposed rule against several terms.

■ Worked example: Sequences

n: 1, 2, 3, 4	n squared: 1, 4, 9, 16	plus 2: 3, 6, 11, 18
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For 4, 7, 10, 13, ..., $a = 4$, $d = 3$, so $u_n = 4 + 3(n-1) = 3n + 1$. The 20th term is $u_{20} = 61$.

- 2, 5, 10, 17, ... is one more than 1, 4, 9, 16, ..., so $u_n = n^2 + 1$.
- 3, 10, 29, 66, ... is two more than the cubes, so $u_n = n^3 + 2$.
- For 1, 3, 6, 10, ..., add 2, 3, 4, then 5 to get 15.

■ Method check

A term-to-term rule and an n th-term rule answer different questions. Test an n th-term expression against the first few terms before using it.

2 Practice

2.1 Write the next two terms of 5, 9, 13, 17, ... [2]

2.2 Find the n th term of 6, 11, 16, 21, ... [2]

2.3 Find the n th term of 3, 6, 11, 18, ... [2]

2.4 Find the fifth term of $u_n = n^3 - 1$. [2]

3 Exam-style questions

3.1 Apply the sequences methods below. Show working and include units where appropriate.

(a) A pattern has 7, 11, 15, 19, ... tiles. Find its n th term. [2]

(b) Find the number of tiles in pattern 30. [2]

(c) Can a pattern in this sequence have 100 tiles? Explain. [2]

3.2 A sequence begins 2, 5, 10, 17, ...

(a) Find the next two terms and an expression for its n th term. [3]

(b) Decide whether 50 and 60 are terms of this sequence. Show how the term number determines each decision. [3]

Solutions

Award method and answer marks as indicated; accept equivalent correct methods.

2.1

- Add 4 each time: **21, 25**.

2.2

- $u_n = 6 + 5(n - 1) = \mathbf{5n + 1}$.

2.3

- Subtract squares 1, 4, 9, 16: the difference is 2, so $u_n = n^2 + 2$.

2.4

- $u_5 = 5^3 - 1 = 125 - 1 = \mathbf{124}$.

3.1

(a)

- $u_n = 7 + 4(n - 1) = \mathbf{4n + 3}$.

(b)

- $u_{30} = 4(30) + 3 = \mathbf{123}$.

(c)

- $4n + 3 = 100$ gives $n = 97/4 = 24.25$. **No**, a pattern number must be a positive integer.

3.2

(a)

- These are one more than successive squares.
- The next terms are **26 and 37**, and $u_n = \mathbf{n^2 + 1}$.

(b)

- For 50, $n^2 = 49$ gives $n = 7$, so **50 is a term**.
- For 60, $n^2 = 59$ lies strictly between 7^2 and 8^2 ; there is no integer index, so **60 is not a term**.