

C2.2 Algebraic manipulation

Name: _____ Class: _____ Date: _____

Total: 20 marks

KEY WORDS term 项 • expand 展开 • area 面积 • like terms 同类项 • brackets 括号

Objective

Use algebraic manipulation to solve Core IGCSE Mathematics problems.

You must be able to:

- Collect like terms and expand brackets
- Expand products of two linear expressions
- Factorise a common factor and check equivalence

1 Key knowledge and worked examples

■ Key knowledge: Algebraic manipulation

Like terms have the same letter part and powers. Add their coefficients only. Expand one bracket by multiplying every term; expand two brackets by making all four products and then collecting. Factorising reverses expansion: take out the greatest factor shared by every term, including letters. Check by expanding your answer.

■ Worked example: Algebraic manipulation

	x	5
x	x^2	$5x$
1	x	5

Expand $(x + 3)(x - 2)$: the four products are $x^2, -2x, 3x, -6$, so $Q = x^2 + x - 6$.

- Collect: $4a + 2b - a + 5b = 3a + 7b$.
- Expand one bracket: $3x(2x - 5) = 6x^2 - 15x$.
- Factorise: $12x^2 + 18x = 6x(2x + 3)$; 6 and x are common to both terms.
- An area is the product of side lengths: for $(x + 2)$ cm and $(x + 3)$ cm, $A = (x + 2)(x + 3) = x^2 + 5x + 6$. At $x = 4$, $A = 16 + 20 + 6 = 42 \text{ cm}^2$.

■ Method check

An area multiplies whole side expressions. The distributive law produces every cross-product; a factorisation should expand back to the original.

2 Practice

2.1 Simplify $8p + 3q - 5p + q$. [2]

2.2 Expand $4y(3y - 2)$. [2]

2.3 Expand $(x + 4)(x + 2)$. [2]

2.4 Factorise fully $15a^2 - 10ab$. [2]

3 Exam-style questions

3.1 Apply the algebraic manipulation methods below. Show working and include units where appropriate.

(a) A rectangle has sides $(x + 5)$ cm and $(x + 1)$ cm. Write and expand its area. [2]

(b) Find its area when $x = 3$. [2]

(c) Factorise $6x^2 + 24x$ fully and check by expanding. [2]

3.2 A rectangle has length $(2x + 3)$ cm and width $(x + 2)$ cm.

(a) Write and expand its area expression. Find the area when $x = 2$. [3]

(b) A student says $6x^2 + 9x = 3x(2x + 9)$. Correct the factorisation and explain how expanding checks it. [3]

Solutions

Award method and answer marks as indicated; accept equivalent correct methods.

2.1

- Collect each letter: $3p + 4q$.

2.2

- Multiply both terms: $12y^2 - 8y$.

2.3

- Four products give $x^2 + 2x + 4x + 8 = \mathbf{x^2 + 6x + 8}$.

2.4

- Take out $5a$: $5a(3a - 2b)$; expanding returns the original.

3.1

(a)

- $A = (x + 5)(x + 1) = \mathbf{x^2 + 6x + 5}$ cm².

(b)

- $A = 3^2 + 6(3) + 5 = \mathbf{32}$ cm²; also $8 \times 4 = 32$.

(c)

- Common factor $6x$ gives $6x(x + 4)$; expansion is $6x^2 + 24x$.

3.2

(a)

- Use $A = lw = (2x + 3)(x + 2)$.
- Expand $A = \mathbf{2x^2 + 7x + 6}$.
- At $x = 2$, $A = 8 + 14 + 6 = \mathbf{28}$ cm².

(b)

- Divide both terms by the common factor $3x$: $6x^2 + 9x = \mathbf{3x(2x + 3)}$.
- Expanding gives $6x^2 + 9x$; the proposed expression instead gives $6x^2 + 27x$.