

C1.7 Indices I

Name: _____ Class: _____ Date: _____

Total: 20 marks**KEY WORDS** base 底数 • power 幂 • index 指数 • rate 比率 • reciprocal 倒数

Objective

Use indices i to solve Core IGCSE Mathematics problems.

You must be able to:

- Use zero and negative integer indices
- Apply product, quotient and power rules
- Separate index laws for products from ordinary addition

1 Key knowledge and worked examples

■ Key knowledge: Indices I

For a nonzero base, $a^0 = 1$ and $a^{-n} = 1/a^n$. Same-base multiplication adds indices; division subtracts them. A power of a power multiplies indices. These rules need the same base: $2^3 \times 3^2$ cannot be combined by adding the indices.

■ Worked example: Indices I

same base

multiply: add

divide:
subtractCalculate $5^{-2} \times 5^3$ and $(2^3)^2$.

- Same-base multiplication: $5^{-2} \times 5^3 = 5^{-2+3} = 5$.
- Power of a power: $(2^3)^2 = 2^{3 \times 2} = 2^6 = 64$.
- A negative index is a reciprocal, not a negative answer: $4^{-1} = 1/4$.

■ Method check

Add indices when multiplying the same base, not when adding terms. A negative index gives a reciprocal; it does not make the result negative.

2 Practice

2.1 Find 9^0 and 2^{-3} . [2]

2.2 Calculate $3^2 \times 3^3$. [2]

2.3 Calculate $2^3 \div 2^5$. [2]

2.4 Calculate $(5^2)^2$. [2]

3 Exam-style questions

3.1 Apply the indices i methods below. Show working and include units where appropriate.

(a) Write $10^4 \times 10^{-2}$ as one power of 10. [2]

(b) Write $10^2 \div 10^5$ as one power of 10. [2]

(c) Calculate $10^4 \times 10^{-2} + 10^2 \div 10^5$. [2]

3.2 Let $A = 2^5 \times 2^{-2}$ and $B = 2^0 + 2^{-2}$.

(a) Calculate A and B, giving B as an exact fraction. [3]

(b) A student writes $A + B = 2^{5-2+0-2}$. Explain the error and calculate the correct total. [3]

Solutions

Award method and answer marks as indicated; accept equivalent correct methods.

2.1

- Zero index gives **1**; $2^{-3} = 1/2^3 = \mathbf{1/8}$.

2.2

- Add indices: $Q = 3^{2+3} = 3^5 = \mathbf{243}$.

2.3

- Subtract indices: $Q = 2^{3-5} = 2^{-2} = \mathbf{1/4}$.

2.4

- Multiply indices: $Q = 5^{2 \times 2} = 5^4 = \mathbf{625}$.

3.1

(a)

- $Q = 10^{4+(-2)} = \mathbf{10^2}$.

(b)

- $R = 10^{2-5} = \mathbf{10^{-3}}$.

(c)

- $S = 100 + 0.001 = \mathbf{100.001}$.

3.2

(a)

- For A, use $a^m a^n = a^{m+n}$: $A = 2^{5-2} = \mathbf{8}$.
- For B, $B = 1 + 1/4 = \mathbf{5/4}$.

(b)

- The rule for multiplying powers does not apply to the plus sign between A and B.
- Use the evaluated values: $S = A + B = 8 + 5/4 = \mathbf{37/4}$.