

C1.3 Powers and roots

Name: _____ Class: _____ Date: _____

Total: 20 marks**KEY WORDS** cube root 立方根 • square root 平方根 • area 面积 • volume 体积 • power 幂

Objective

Use powers and roots to solve Core IGCSE Mathematics problems.

You must be able to:

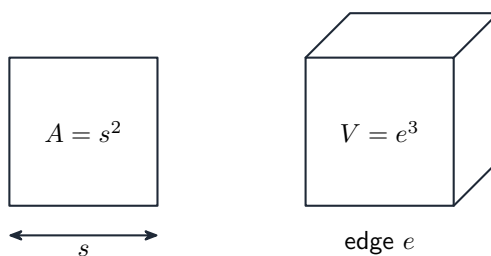
- Evaluate squares, cubes and their roots
- Distinguish a negative base from a negative sign outside a power
- Recover a positive length from an area or volume

1 Key knowledge and worked examples

■ Key knowledge: Powers and roots

A power repeats multiplication, not addition. $a^2 = a \times a$ and $a^3 = a \times a \times a$. The square root is the nonnegative number whose square is the given value. A cube root can be negative. Learn squares from 1 to 15 and cubes of 1, 2, 3, 4, 5 and 10. Brackets decide whether a minus sign is part of the base.

■ Worked example: Powers and roots

Find $(-3)^2 + \sqrt[3]{64}$.

- Square the whole bracket: $(-3)^2 = (-3)(-3) = 9$.
- Since $4^3 = 64$, $\sqrt[3]{64} = 4$.
- Thus $Q = (-3)^2 + \sqrt[3]{64} = 9 + 4 = 13$. By contrast, $-3^2 = -(3^2) = -9$.

■ Method check

Brackets decide whether a minus sign is squared: $(-a)^2$ is positive, while $-a^2$ is the negative of a square. A physical side length is positive.

2 Practice

2.1 Find 12^2 and $\sqrt{225}$. [2]

2.2 Find 5^3 and $\sqrt[3]{-27}$. [2]

2.3 Calculate 3^4 and $\sqrt[4]{16}$. [2]

2.4 Calculate $2^3 \times \sqrt{49}$. [2]

3 Exam-style questions

3.1 Apply the powers and roots methods below. Show working and include units where appropriate.

- (a) A square has area 196 cm^2 . Find its side length. [2]

- (b) A cube has volume 1000 cm^3 . Find its edge length. [2]

- (c) Find the sum of the square perimeter and the total length of the 12 cube edges. [2]

3.2 A square tile has area 144 cm^2 . A cube has volume 216 cm^3 .

- (a) Find the tile side and cube edge, stating which root is needed for each. [3]

- (b) A student claims that $(-4)^2 = -4^2$. Calculate both expressions and explain the role of the brackets. [3]

Solutions

Award method and answer marks as indicated; accept equivalent correct methods.

2.1

- $12^2 = 12 \times 12 = \mathbf{144}$; $15^2 = 225$, so $\sqrt{225} = \mathbf{15}$.

2.2

- $5^3 = 5 \times 5 \times 5 = \mathbf{125}$; $(-3)^3 = -27$, so the cube root is -3 .

2.3

- Four factors give $3^4 = \mathbf{81}$. Since $2^4 = 16$, the positive fourth root is $\mathbf{2}$.

2.4

- Powers and roots first: $Q = 8 \times 7 = \mathbf{56}$.

3.1

(a)

- $s^2 = A$, so $s = \sqrt{196} = \mathbf{14}$ cm.

(b)

- $e^3 = V$, so $e = \sqrt[3]{1000} = \mathbf{10}$ cm.

(c)

- $L = 4s + 12e = 4(14) + 12(10) = \mathbf{176}$ cm.

3.2

(a)

- A square has $A = s^2$, so $s = \sqrt{A} = \sqrt{144} = \mathbf{12}$ cm.
- A cube has $V = e^3$, so $e = \sqrt[3]{216} = \mathbf{6}$ cm; square root for area, cube root for volume.

(b)

- $(-4)^2 = (-4)(-4) = \mathbf{16}$, while $-4^2 = -(4 \times 4) = \mathbf{-16}$.
- The brackets include the negative sign in the base; without them, the power is evaluated first.