

C1.12 Rates

Name: _____ Class: _____ Date: _____

Total: 20 marks

KEY WORDS average speed 平均速度 • density 密度 • population density 人口密度 • rate 比率
• area 面积

Objective

Use rates to solve Core IGCSE Mathematics problems.

You must be able to:

- Calculate and rearrange distance, time and rate relationships
- Keep rate units consistent
- Include stopped time in a whole-journey average

1 Key knowledge and worked examples

■ Key knowledge: Rates

A rate compares quantities per unit, such as yuan per hour. Multiply rate by amount to find a total; divide total by rate to find the amount. Average speed is total distance divided by total time, including stops. Convert minutes to hours by dividing by 60. For unfamiliar rates, use the formula given in the question.

■ Worked example: Rates

distance: 42 km	time: 7/3 h	speed = distance/time
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A cyclist covers 42 km in 2 hours 20 minutes.

- Convert time: $t = 2 + 20/60 = 7/3$ h.
- Use $v = d/t$ because speed is distance per hour.
- $v = 42/(7/3) = 18$ km/h.
- If density is $\rho = m/V$, then $\rho = 90/30 = 3$ g/cm³.

■ Method check

Average speed is total distance divided by total elapsed time. It is not usually the average of the speeds of the separate stages.

2 Practice

2.1 A worker earns 32 yuan per hour. Find the pay for 7.5 hours. [2]

2.2 A tap supplies 12 litres per minute. Find the time to fill 90 litres. [2]

2.3 A car travels 156 km in 2 hours 36 minutes. Find its average speed. [2]

2.4 Population density is population divided by area. Find it for 8400 people in 12 km^2 . [2]

3 Exam-style questions

3.1 Apply the rates methods below. Show working and include units where appropriate.



135 km uses 9 litres

(a) A van uses 9 litres to travel 135 km. Find its fuel use in litres per 100 km. [2]

(b) At this rate, find the fuel needed for 240 km. [2]

(c) The 240 km trip takes 3 hours driving plus 1 hour stopped. Find the average speed for the whole trip. [2]

3.2 A cyclist travels 18 km in 45 min, rests for 15 min, then travels 12 km in 40 min.

(a) Find the speed in each moving stage in km/h and identify the faster stage. [3]

(b) Find the average speed for the whole trip. Explain why 21 km/h is not the correct whole-trip average. [3]

Solutions

Award method and answer marks as indicated; accept equivalent correct methods.

2.1

- $P = rt = 32 \times 7.5 = \mathbf{240}$ yuan.

2.2

- $t = V/r = 90/12 = \mathbf{7.5}$ minutes.

2.3

- $t = 2 + 36/60 = 2.6$ h; $v = d/t = 156/2.6 = \mathbf{60}$ km/h.

2.4

- $D = P/A = 8400/12 = \mathbf{700}$ people per km².

3.1

(a)

- $r = (9/135) \times 100 = \mathbf{20/3}$ litres per 100 km (about 6.67).

(b)

- $F = (9/135) \times 240 = \mathbf{16}$ litres.

(c)

- $t = 3 + 1 = 4$ h; $v = d/t = 240/4 = \mathbf{60}$ km/h.

3.2

(a)

- Convert times to hours and use $v = d/t$.
- $v_1 = 18/(45/60) = \mathbf{24}$ and $v_2 = 12/(40/60) = \mathbf{18}$ km/h.
- The first stage is faster.

(b)

- Total time includes the rest: $t = (45 + 15 + 40)/60 = 5/3$ h.
- $v_{\text{avg}} = 30/(5/3) = \mathbf{18}$ km/h.
- The value 21 simply averages stage speeds and ignores the rest and unequal stage times.