

Exercise walk-through

E9.3

Averages and measures of spread

IGCSE Mathematics

You must be able to

- find the mean, median, mode and range of a list of values
- find the quartiles and the interquartile range
- estimate the mean from a grouped frequency table, and say why it is only an estimate

1

The method

Method — The median needs the list in order, every time

- **Mean** = total $\div$ how many. **Median** is the middle value of the ORDERED list. **Mode** is the most common value.
- **Range** = largest – smallest. **IQR** = $Q_3 - Q_1$, which ignores the extremes and so is less affected by an outlier.
- **Estimated mean from a grouped table:** use the MIDPOINT of each class, then
$$\frac{\sum fx}{\sum f}.$$
- **Why it is an estimate:** the exact values inside each class are unknown, and the midpoint assumes they are evenly spread.

Answer shapes (marked by **method marks** — M1 for a correct method, A1 for the correct answer): the midpoints row, and the $\sum fx$ total, are separate method marks in an estimated-mean question.

Method — The median needs the list in order, every time

Answer shapes (marked by method marks — M1 for a correct method, A1 for the correct answer): the midpoints...

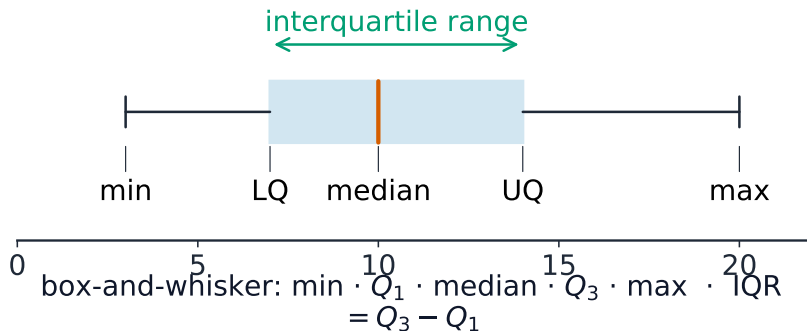


$$\begin{aligned}\text{median} &= 3 \\ \text{mean} &= \frac{2+3+3+5+7}{5} = 4 \\ \text{mode} &= 3 \text{ (most common)} \\ \text{range} &= 7 - 2 = 5\end{aligned}$$

The mean, median and mode of the same data set

Method — The median needs the list in order, every time

Answer shapes (marked by method marks — M1 for a correct method, A1 for the correct answer): the midpoints...



A box-and-whisker diagram showing the quartiles

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Practice

Practice 2.1 ■ 4 marks

Find the mean, median, mode and range of 4, 7, 7, 9, 13.

Solution 2.1

Method: The median needs the list in order, every time — p.4

Worked steps

- $\text{mean} = \frac{4 + 7 + 7 + 9 + 13}{5} = \frac{40}{5} = 8$ (B1)
- the list is already in order, so the median is the third value, 7 (B1)
- mode = 7, the only repeated value (B1)
- range = $13 - 4 = 9$ (B1)

Practice 2.2 ■ 3 marks

Find the median and the interquartile range of
3, 5, 8, 8, 11, 14, 17, 21.

Solution 2.2

Method: The median needs the list in order, every time — p.4

Worked steps

■ eight values, so the median is the mean of the 4th and 5th: $\frac{8 + 11}{2} = \mathbf{9.5}$ **M1** **A1**

■ $Q_1 = \frac{5 + 8}{2} = 6.5$ and $Q_3 = \frac{14 + 17}{2} = 15.5$

$$\text{IQR} = 15.5 - 6.5 = \mathbf{9}$$

A1

Practice 2.3 ■ 1 mark

The mean of six numbers is 14. Find their total.

Solution 2.3

Method: The median needs the list in order, every time — p.4

Worked steps

■ $\text{total} = 14 \times 6$

$$= 84$$

B1

3

Exam-style

Exam-style 3.1 ■ 6 marks

The table shows the times t minutes taken by 30 students.

Time (t minutes)	Frequency
$0 < t \leq 10$	4
$10 < t \leq 20$	11
$20 < t \leq 30$	9
$30 < t \leq 40$	6

Exam-style 3.1 (a) ■ 4 marks

Calculate an estimate of the mean time, correct to 3 significant figures.

Solution 3.1 (a)

Method: The median needs the list in order, every time — p.4

Worked steps

■ midpoints 5, 15, 25, 35 (M1)

■ $\sum fx = 4(5) + 11(15) + 9(25) + 6(35) = 20 + 165 + 225 + 210 = 620$ (M1) (M1)

■ $\sum f = 30$

$$\frac{620}{30} = 20.67 \dots = \mathbf{20.7} \text{ minutes} \quad (\text{A1})$$

Exam-style 3.1 (b) ■ 1 mark

Write down the modal class.

Solution 3.1 (b)

Worked steps

- the class with the highest frequency, 11

$$10 < t \leq 20$$

B1

- all four classes have the same width here, so the highest frequency is also the highest frequency density

Exam-style 3.1 (c) ■ 1 mark

Explain why the mean can only be an estimate.

Solution 3.1 (c)

Worked steps

- the individual times inside each class are not known; the midpoint assumes they are spread evenly through the class **B1**

Exam-style 3.2 ■ 3 marks

The mean of five numbers is 12. A sixth number is added and the mean becomes 13. Find the sixth number.

Solution 3.2

Worked steps

- the first five total $12 \times 5 = 60$ **M1**
- all six total $13 \times 6 = 78$ **M1**

$$78 - 60 = 18$$

A1