

Exercise walk-through

E8.3

Probability of combined events

IGCSE Mathematics

You must be able to

- draw and complete a tree diagram for two events
- multiply along branches and add between branches
- change the denominators correctly when there is no replacement

1

The method

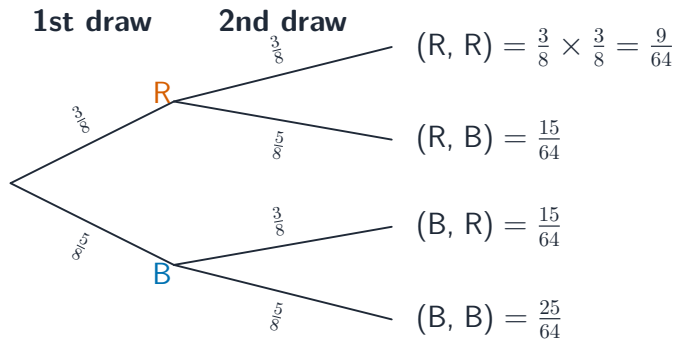
Method — Multiply along a branch, add between branches

- **AND** $\Rightarrow$ **multiply**, **OR** $\Rightarrow$ **add**. $P(\text{both red})$ is one path; $P(\text{one of each})$ is TWO paths added.
- **With replacement**: the second set of probabilities is the same as the first.
- **Without replacement**: BOTH the top and the bottom of the second fraction fall. From 5 red in 8, the second red is $\frac{4}{7}$, not $\frac{5}{8}$ and not $\frac{5}{7}$.
- **“At least one”**: it is almost always faster to use $1 - P(\text{none})$.

Answer shapes (marked by **method marks** — M1 for a correct method, A1 for the correct answer): each correct product is a method mark, and adding the right number of them is another.

Method — Multiply along a branch, add between branches

Answer shapes (marked by method marks — M1 for a correct method, A1 for the correct answer): each correct...



A two-stage tree diagram with probabilities on each branch

Method — Multiply along a branch, add between branches

Answer shapes (marked by method marks — M1 for a correct method, A1 for the correct answer): each correct...

		die 2					
+		1	2	3	4	5	6
die 1	1	2	3	4	5	6	7
	2	3	4	5	6	7	8
	3	4	5	6	7	8	9
	4	5	6	7	8	9	10
	5	6	7	8	9	10	11
	6	7	8	9	10	11	12

the six highlighted cells all give a total of 7

A sample space diagram for two events

2

Practice

Practice 2.1 ■ 2 marks

Two fair coins are flipped. Find the probability that both land on heads.

Solution 2.1

Method: Multiply along a branch, add between branches — p.4

Worked steps

$$\blacksquare \frac{1}{2} \times \frac{1}{2} \quad \text{M1}$$

$$= \frac{1}{4}$$

A1

Practice 2.2 ■ 2 marks

A bag contains 5 red and 3 blue counters. Two are taken at random **with** replacement. Find the probability that both are red.

Solution 2.2

Method: Multiply along a branch, add between branches — p.4

Worked steps

■ with replacement the probabilities are unchanged: $\frac{5}{8} \times \frac{5}{8}$ **M1**

$$= \frac{25}{64}$$

A1

Practice 2.3 ■ 3 marks

Repeat for the same bag, this time **without** replacement.

Solution 2.3

Method: Multiply along a branch, add between branches — p.4

Worked steps

- after one red is removed, 4 reds remain out of 7 counters M1

- $\frac{5}{8} \times \frac{4}{7} = \frac{20}{56}$ M1

$$= \frac{5}{14}$$

A1

3

Exam-style

Exam-style 3.1 ■ 9 marks

A bag contains 7 green and 5 yellow counters. Two counters are taken at random without replacement.

Exam-style 3.1 (a) ■ 3 marks

Find the probability that both are green.

Solution 3.1 (a)

Method: Multiply along a branch, add between branches — p.4

Worked steps

$$\blacksquare \frac{7}{12} \times \frac{6}{11} \quad \text{M1} \quad \text{M1}$$

$$= \frac{42}{132} = \frac{7}{22}$$

A1

Exam-style 3.1 (b) ■ 4 marks

Find the probability that one is green and one is yellow.

Solution 3.1 (b)

Worked steps

- green then yellow: $\frac{7}{12} \times \frac{5}{11} = \frac{35}{132}$ **M1**
- yellow then green: $\frac{5}{12} \times \frac{7}{11} = \frac{35}{132}$ **M1**
- there are TWO orders, so add them **M1**

$$\frac{70}{132} = \frac{35}{66}$$

A1

Exam-style 3.1 (c) ■ 2 marks

Find the probability that at least one is yellow.

Solution 3.1 (c)

Worked steps

- “at least one yellow” is the opposite of “both green” M1

$$1 - \frac{7}{22} = \frac{15}{22}$$

A1

Exam-style 3.2 ■ 3 marks

The probability that it rains on any given day is 0.3, independently of other days. Find the probability that it rains on exactly one of two days.

Solution 3.2

Worked steps

■ rain then dry: $0.3 \times 0.7 = 0.21$ **M1**

■ dry then rain: $0.7 \times 0.3 = 0.21$ **M1**

$$0.21 + 0.21 = \mathbf{0.42} \quad \mathbf{A1}$$

- forgetting the second order and answering 0.21 is the standard loss here