Exercise walk-through

E7.4
Vector geometry

IGCSE Mathematics

You must be able to

- write any journey in a diagram in terms of the given vectors
- find the position vector of a midpoint or of a point dividing a line in a given ratio
- prove two vectors are parallel, and prove three points are collinear

1 The method

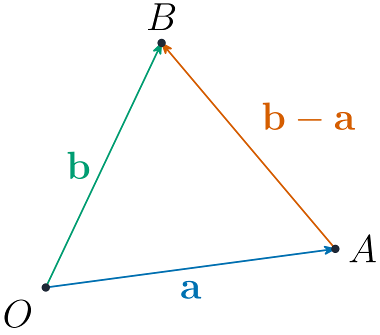
Method — Travel there by any route — the answer is the same

- **The basic move:** $\overrightarrow{AB} = \mathbf{b} - \mathbf{a}$ when $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$. "Destination minus start".
- **Midpoint:** $\overrightarrow{OM} = \mathbf{a} + \frac{1}{2}(\mathbf{b} - \mathbf{a}) = \frac{1}{2}(\mathbf{a} + \mathbf{b})$.
- **Parallel:** two vectors are parallel when one is a **scalar multiple** of the other. $4\mathbf{a} + 6\mathbf{b} = 2(2\mathbf{a} + 3\mathbf{b})$, so those two are parallel.
- **Collinear:** parallel AND sharing a point. $\overrightarrow{OQ} = 3\overrightarrow{OP}$ with the common point O means O, P and Q lie on one straight line.

Answer shapes (marked by **method marks** — M1 for a correct method, A1 for the correct answer): a valid route written down is the method mark, even before it is simplified. For a proof, the words "scalar multiple" and "common point" are the marks.

Method — Travel there by any route — the answer is the same

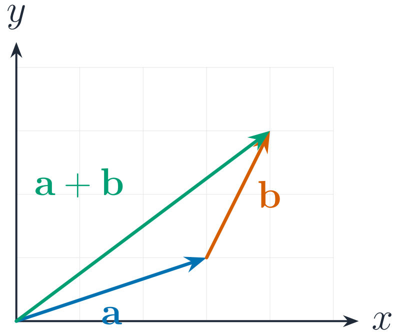
Answer shapes (marked by method marks — M1 for a correct method, A1 for the correct answer): a valid route...



A vector journey around a shape, written in terms of two base vectors

Method — Travel there by any route — the answer is the same

Answer shapes (marked by method marks — M1 for a correct method, A1 for the correct answer): a valid route...



Two vectors added nose to tail

2 Practice

Practice 2.1 ▪ 1 mark

$\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$. Write $\overrightarrow{AB}$ in terms of $\mathbf{a}$ and $\mathbf{b}$.

Solution 2.1

Method: Travel there by any route — the answer is the same — p.4

Worked steps

- $\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB}$

$$= \mathbf{b} - \mathbf{a}$$ B1

Practice 2.2 ▪ 2 marks

M is the midpoint of AB. Write $\overrightarrow{OM}$ in terms of $\mathbf{a}$ and $\mathbf{b}$, in its simplest form.

Solution 2.2

Method: Travel there by any route — the answer is the same — p.4

Worked steps

- $\overrightarrow{OM} = \mathbf{a} + \frac{1}{2}(\mathbf{b} - \mathbf{a})$ **M1**

$$= \frac{\mathbf{1}}{\mathbf{2}}(\mathbf{a} + \mathbf{b})$$ **A1**

Practice 2.3 ▪ 2 marks

$\overrightarrow{PQ} = 2\mathbf{a} + 3\mathbf{b}$ and $\overrightarrow{RS} = 4\mathbf{a} + 6\mathbf{b}$. Show that PQ and RS are parallel.

Solution 2.3

Method: Travel there by any route — the answer is the same — p.4

Worked steps

- $\overrightarrow{RS} = 2\,(2\mathbf{a} + 3\mathbf{b}) = 2\,\overrightarrow{PQ}$ **M1**
- one is a scalar multiple of the other, so they are parallel **A1**

3

Exam-style

Exam-style 3.1 ▪ 6 marks

$OABC$ is a parallelogram with $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OC} = \mathbf{c}$. M is the midpoint of AB.

Exam-style 3.1 (a) ▪ 1 mark

Write down $\overrightarrow{AB}$ in terms of $\mathbf{c}$.

Solution 3.1 (a)

Method: Travel there by any route — the answer is the same — p.4

Worked steps

- in a parallelogram AB is parallel to and equal in length to OC

$$= \mathbf{c}$$

B1

Exam-style 3.1 (b) ▪ 2 marks

Find $\overrightarrow{OM}$ in terms of $\mathbf{a}$ and $\mathbf{c}$.

Solution 3.1 (b)

Worked steps

- $\overrightarrow{OM} = \overrightarrow{OA} + \frac{1}{2}\overrightarrow{AB}$ **M1**

$$= \mathbf{a} + \frac{\mathbf{1}}{\mathbf{2}}\mathbf{c}$$ **A1**

Exam-style 3.1 (c) ▪ 3 marks

N lies on OB with $\overrightarrow{ON} = \frac{2}{3}\overrightarrow{OB}$. Find $\overrightarrow{MN}$ in terms of $\mathbf{a}$ and $\mathbf{c}$.

Solution 3.1 (c)

Worked steps

- $\overrightarrow{OB} = \mathbf{a} + \mathbf{c}$, so $\overrightarrow{ON} = \frac{2}{3}\mathbf{a} + \frac{2}{3}\mathbf{c}$ **M1**
- $\overrightarrow{MN} = \overrightarrow{ON} - \overrightarrow{OM} = \left(\frac{2}{3}\mathbf{a} + \frac{2}{3}\mathbf{c}\right) - \left(\mathbf{a} + \frac{1}{2}\mathbf{c}\right)$ **M1**

$$= -\frac{\mathbf{1}}{\mathbf{3}}\mathbf{a} + \frac{\mathbf{1}}{\mathbf{6}}\mathbf{c}$$ **A1**

Exam-style 3.2 ▪ 3 marks

$\overrightarrow{OP} = 3\mathbf{a} - \mathbf{b}$ and $\overrightarrow{OQ} = 9\mathbf{a} - 3\mathbf{b}$. Show that O, P and Q are collinear.

Solution 3.2

Method: Travel there by any route — the answer is the same — p.4

Worked steps

- $\overrightarrow{OQ} = 3\,(3\mathbf{a} - \mathbf{b}) = 3\,\overrightarrow{OP}$ **M1**
- so $\overrightarrow{OQ}$ is a scalar multiple of $\overrightarrow{OP}$, i.e. the two are parallel **M1**
- they share the point O, so O, P and Q are **collinear** **A1**
- parallel alone is not enough; the common point is what makes it one straight line