Exercise walk-through

E7.3
Magnitude of a vector

IGCSE Mathematics

You must be able to

- find the magnitude of a column vector using Pythagoras
- add, subtract and multiply column vectors before finding a magnitude
- find a missing component when the magnitude is given

1 The method

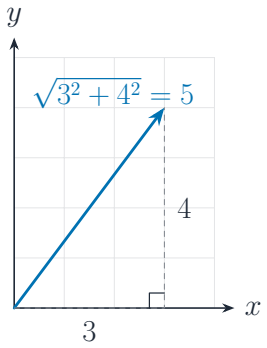
Method — Magnitude is Pythagoras on the two components

- **Formula:** if $\mathbf{a} = \begin{pmatrix} x \\ y \end{pmatrix}$ then $|\mathbf{a}| = \sqrt{x^2 + y^2}$.
- **The signs vanish**, because both components are squared. $\begin{pmatrix} 6 \\ -8 \end{pmatrix}$ has magnitude 10, the same as $\begin{pmatrix} 6 \\ 8 \end{pmatrix}$.
- **Combine FIRST.** $|\mathbf{a} + \mathbf{b}|$ means add the vectors and then take the magnitude; it is NOT $|\mathbf{a}| + |\mathbf{b}|$.
- **Scalar multiple:** $2\mathbf{a}$ multiplies BOTH components by 2.

Answer shapes (marked by **method marks** — M1 for a correct method, A1 for the correct answer): the combined column vector, and the square root expression, are separate method marks.

Method — Magnitude is Pythagoras on the two components

Answer shapes (marked by method marks — M1 for a correct method, A1 for the correct answer): the combined...



A column vector drawn on a grid with its magnitude marked

2 Practice

Practice 2.1 ▪ 2 marks

$\mathbf{a} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$. Find $|\mathbf{a}|$.

Solution 2.1

Method: Magnitude is Pythagoras on the two components — p.4

Worked steps

- $\sqrt{3^2 + 4^2} = \sqrt{25}$ M1

$$= \mathbf{5}$$ A1

Practice 2.2 ▪ 2 marks

$\mathbf{b} = \begin{pmatrix} -5 \\ 12 \end{pmatrix}$. Find $|\mathbf{b}|$.

Solution 2.2

Method: Magnitude is Pythagoras on the two components — p.4

Worked steps

- $\sqrt{(-5)^2 + 12^2} = \sqrt{169}$ M1

$$= \mathbf{13}$$ A1

Practice 2.3 ▪ 2 marks

$\mathbf{c} = \begin{pmatrix} 6 \\ -8 \end{pmatrix}$. Find $|\mathbf{c}|$.

Solution 2.3

Method: Magnitude is Pythagoras on the two components — p.4

Worked steps

- $\sqrt{6^2 + (-8)^2} = \sqrt{100}$ M1

$$= \mathbf{10}$$ A1

3 Exam-style

Exam-style 3.1 ▪ 6 marks

$\mathbf{a} = \begin{pmatrix} 4 \\ -3 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} -1 \\ 5 \end{pmatrix}$.

Exam-style 3.1 (a) ▪ 1 mark

Find $\mathbf{a} + \mathbf{b}$ as a column vector.

Solution 3.1 (a)

Method: Magnitude is Pythagoras on the two components — p.4

Worked steps

- add the components separately

$$\begin{pmatrix} 3 \\ 2 \end{pmatrix}$$

B1

Exam-style 3.1 (b) ▪ 2 marks

Find $|\mathbf{a} + \mathbf{b}|$, giving your answer in exact form.

Solution 3.1 (b)

Worked steps

- $\sqrt{3^2 + 2^2} = \sqrt{13}$ M1

  $= \sqrt{\mathbf{13}}$ A1

- 13 has no square factor, so this is already exact

Exam-style 3.1 (c) ▪ 3 marks

Find $|2\mathbf{a} - \mathbf{b}|$, correct to 3 significant figures.

Solution 3.1 (c)

Worked steps

- $2\mathbf{a} = \begin{pmatrix} 8 \\ -6 \end{pmatrix}$, so $2\mathbf{a} - \mathbf{b} = \begin{pmatrix} 9 \\ -11 \end{pmatrix}$ M1 A1
- $\sqrt{81 + 121} = \sqrt{202}$

$$= 14.212\ldots = \mathbf{14.2}$$ A1

Exam-style 3.2 ▪ 3 marks

The vector $\begin{pmatrix} k \\ 8 \end{pmatrix}$ has magnitude 17. Find the positive value of k.

Solution 3.2

Method: Magnitude is Pythagoras on the two components — p.4

Worked steps

- $k^2 + 8^2 = 17^2$ M1
- $k^2 = 289 - 64 = 225$ M1

$$k = \mathbf{15}$$ A1