Exercise walk-through

E6.6
Pythagoras' theorem and trigonometry in 3D

IGCSE Mathematics

You must be able to

- find a length in three dimensions by using Pythagoras twice
- identify the right-angled triangle inside a 3D solid and draw it separately
- find the angle between an edge and a base

1 The method

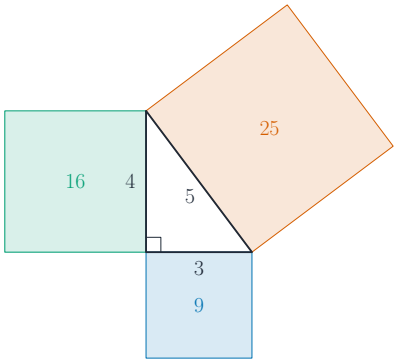
Method — Draw the 2D triangle out of the 3D picture

- **Two steps, always:** find a length across the BASE first, then use it with the vertical height.
- **Cuboid space diagonal:** $\sqrt{l^2 + w^2 + h^2}$ — which is just Pythagoras done twice, with the base diagonal in the middle.
- **Angle with the base:** the right angle is between the vertical height and the base, so $\tan\theta = \dfrac{\text{height}}{\text{horizontal distance}}$.
- **Never round the intermediate length.** Keep it in the calculator; rounding the base diagonal first is what costs the final accuracy mark.

Answer shapes (marked by **method marks** — M1 for a correct method, A1 for the correct answer): the first Pythagoras and the second are separate method marks, so show both lines.

Method — Draw the 2D triangle out of the 3D picture

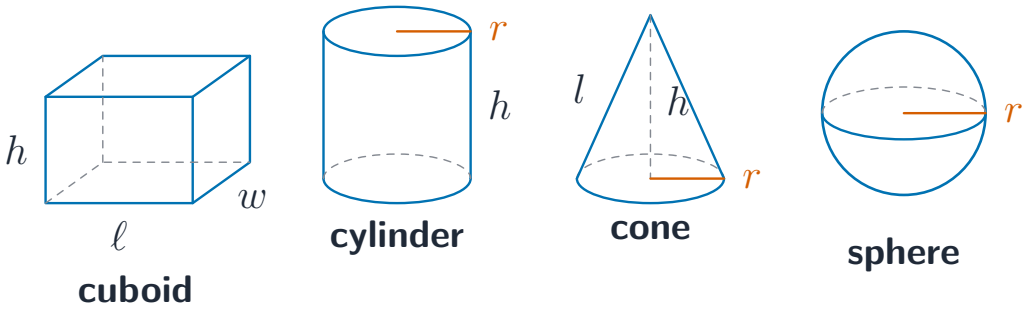
Answer shapes (marked by method marks — M1 for a correct method, A1 for the correct answer): the first...



A right-angled triangle with the sides labelled for Pythagoras

Method — Draw the 2D triangle out of the 3D picture

Answer shapes (marked by method marks — M1 for a correct method, A1 for the correct answer): the first...



The standard solids and their dimensions

2 Practice

Practice 2.1 ▪ 2 marks

A cuboid measures 6 cm by 8 cm by 10 cm. Find the length of the diagonal of the 6 cm by 8 cm face.

Solution 2.1

Method: Draw the 2D triangle out of the 3D picture — p.4

Worked steps

- $\sqrt{6^2 + 8^2} = \sqrt{100}$ M1

$= \mathbf{10}$ cm A1

Practice 2.2 ▪ 3 marks

Find the length of the space diagonal of that cuboid, correct to 3 significant figures.

Solution 2.2

Method: Draw the 2D triangle out of the 3D picture — p.4

Worked steps

- use the base diagonal 10 cm with the height 10 cm M1
- $\sqrt{10^2 + 10^2} = \sqrt{200}$ M1

$$= 14.142\ldots = \mathbf{14.1}\text{ cm} \quad \text{A1}$$

- equivalently $\sqrt{6^2 + 8^2 + 10^2}$ in one line

Practice 2.3 ▪ 3 marks

Find the angle between the space diagonal and the 6 cm by 8 cm base, correct to 1 decimal place.

Solution 2.3

Method: Draw the 2D triangle out of the 3D picture — p.4

Worked steps

- the triangle has the base diagonal 10 cm and the height 10 cm, with the right angle between them M1
- $\tan\theta = \dfrac{10}{10} = 1$ M1

$$\theta = \mathbf{45.0}^\circ$$ A1

3 Exam-style

Exam-style 3.1 ▪ 9 marks

A pyramid has a square base of side 10 cm and a vertical height of 12 cm from the centre of the base to the apex.

Exam-style 3.1 (a) ▪ 3 marks

Find the distance from the centre of the base to one corner of the base, correct to 3 significant figures.

Solution 3.1 (a)

Method: Draw the 2D triangle out of the 3D picture — p.4

Worked steps

- the base diagonal $= \sqrt{10^2 + 10^2} = \sqrt{200} = 14.142\ldots$ cm M1 M1
- the centre is halfway along it

$$14.142 \div 2 = \mathbf{7.07} \text{ cm} \quad \text{A1}$$

Exam-style 3.1 (b) ▪ 3 marks

Find the length of a slant edge, correct to 3 significant figures.

Solution 3.1 (b)

Worked steps

- the slant edge, the height and the half-diagonal form a right-angled triangle M1
- slant $= \sqrt{12^2 + 7.071^2} = \sqrt{194}$ M1

$$= 13.928\ldots = \mathbf{13.9}\text{ cm}$$ A1

Exam-style 3.1 (c) ▪ 3 marks

Find the angle between a slant edge and the base, correct to 1 decimal place.

Solution 3.1 (c)

Worked steps

- $\tan\theta = \dfrac{12}{7.071}$ M1 M1

$$\theta = 59.49\ldots = \mathbf{59.5}^\circ \quad \text{A1}$$

- the horizontal side is the HALF-diagonal, not the full one and not the side of the square