

Exercise walk-through

E6.5

Non-right-angled triangles

IGCSE Mathematics

You must be able to

- choose the correct rule from what the question gives you
- use the sine rule to find a side or an angle
- use the cosine rule to find a third side, and use $\frac{1}{2}ab \sin C$ for an area

1

The method

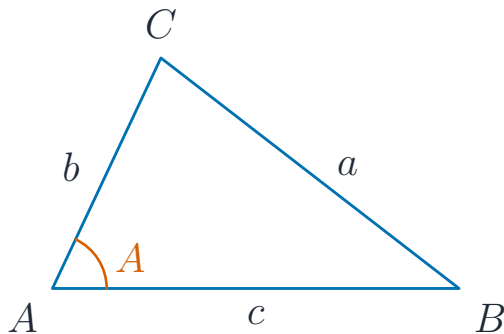
Method — Which rule? Count what you are given

- **Sine rule:** $\frac{a}{\sin A} = \frac{b}{\sin B}$. Use it when you have a side with its OPPOSITE angle, plus one more piece.
- **Cosine rule:** $a^2 = b^2 + c^2 - 2bc \cos A$. Use it for two sides and the angle BETWEEN them, or for all three sides.
- **Area:** $\frac{1}{2}ab \sin C$, where C is the angle between the two sides a and b .
- **The ambiguous case:** when the sine rule gives an angle, 180° minus that angle also has the same sine. Check which fits the triangle.

Answer shapes (marked by **method marks** — M1 for a correct method, A1 for the correct answer): quoting the rule with the numbers substituted is the method mark, and the rearranged line is a second one.

Method — Which rule? Count what you are given

Answer shapes (marked by method marks — M1 for a correct method, A1 for the correct answer): quoting the rule...



A triangle labelled for the cosine rule

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Practice

Practice 2.1 ■ 3 marks

In triangle ABC , $a = 9$ cm, $A = 40^\circ$ and $B = 65^\circ$. Find b , correct to 3 significant figures.

Solution 2.1

Method: Which rule? Count what you are given — p.4

Worked steps

■ sine rule: $\frac{b}{\sin 65^\circ} = \frac{9}{\sin 40^\circ}$ (M1)

■ $b = \frac{9 \sin 65^\circ}{\sin 40^\circ}$ (M1)

$$= 12.689 \dots = \mathbf{12.7 \text{ cm}}$$

(A1)

Practice 2.2 ■ 3 marks

In triangle PQR , $p = 7$ cm, $q = 9$ cm and $R = 52^\circ$. Find r , correct to 3 significant figures.

Solution 2.2

Method: Which rule? Count what you are given — p.4

Worked steps

■ cosine rule, since the angle lies between the two known sides **M1**

■ $r^2 = 7^2 + 9^2 - 2(7)(9) \cos 52^\circ = 130 - 126 \cos 52^\circ$ **M1**

■ $r^2 = 52.43 \dots$

$$r = 7.24 \text{ cm} \quad \textbf{A1}$$

■ take the square root at the very end, never of the separate terms

Practice 2.3 ■ 2 marks

Find the area of a triangle with sides 10 cm and 14 cm and an included angle of 38° , correct to 3 significant figures.

Solution 2.3

Method: Which rule? Count what you are given — p.4

Worked steps

$$\blacksquare \text{ area} = \frac{1}{2} \times 10 \times 14 \times \sin 38^\circ \quad \text{M1}$$

$$= 43.096 \dots = \mathbf{43.1 \text{ cm}^2} \quad \text{A1}$$

3

Exam-style

Exam-style 3.1 ■ 5 marks

In triangle ABC , $AB = 8$ cm, $BC = 11$ cm and angle $ABC = 73^\circ$.

Exam-style 3.1 (a) ■ 3 marks

Find AC , correct to 3 significant figures.

Solution 3.1 (a)

Method: Which rule? Count what you are given — p.4

Worked steps

- the angle is between the two known sides, so use the cosine rule **M1**
- $AC^2 = 8^2 + 11^2 - 2(8)(11) \cos 73^\circ = 185 - 176 \cos 73^\circ$ **M1**
- $AC^2 = 133.54 \dots$

$$AC = 11.6 \text{ cm}$$

A1

Exam-style 3.1 (b) ■ 2 marks

Find the area of the triangle, correct to 3 significant figures.

Solution 3.1 (b)

Worked steps

$$\blacksquare \text{ area} = \frac{1}{2} \times 8 \times 11 \times \sin 73^\circ \quad \text{M1}$$

$$= 42.077 \dots = \mathbf{42.1 \text{ cm}^2} \quad \text{A1}$$

Exam-style 3.2 ■ 3 marks

In triangle XYZ , $x = 6.2$ cm, $y = 9.4$ cm and angle $X = 35^\circ$. Find the acute value of angle Y , correct to 1 decimal place.

Solution 3.2

Method: Which rule? Count what you are given — p.4

Worked steps

■ sine rule: $\frac{\sin Y}{9.4} = \frac{\sin 35^\circ}{6.2}$ (M1)

■ $\sin Y = \frac{9.4 \sin 35^\circ}{6.2} = 0.8696 \dots$ (M1)

$$Y = 60.4^\circ$$

(A1)

- the obtuse solution $180 - 60.4 = 119.6^\circ$ also has this sine; the question asked for the acute value