

Exercise walk-through

E6.3

Exact trigonometric values

IGCSE Mathematics

You must be able to

- write down the exact value of $\sin$, $\cos$ and $\tan$ for the standard angles
- use an exact value to give a length or an area in surd form
- solve a simple trigonometric equation over a given range

1

The method

Method — Two triangles give every exact value you need

θ	$\sin \theta$	$\cos \theta$	$\tan \theta$
0°	0	1	0
30°	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$
45°	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1
60°	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
90°	1	0	undefined

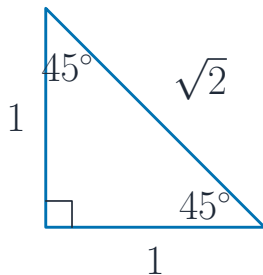
Method — Two triangles give every exact value you need

- **Notice the symmetry:** $\sin \theta$ and $\cos \theta$ swap as θ goes from 30° to 60° , and $\tan 45^\circ = 1$ because the two short sides are equal.
- **“Exact” means no decimals.** $\frac{\sqrt{3}}{2}$, not 0.866.
- **Two answers:** $\sin$ is positive in the first AND second quadrant, so $\sin x = \frac{\sqrt{2}}{2}$ has solutions 45° and 135° in $0^\circ \leq x \leq 180^\circ$.

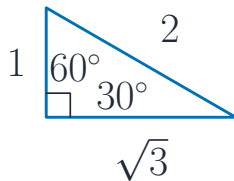
Answer shapes (marked by **method marks** — M1 for a correct method, A1 for the correct answer): quoting the exact value is a method mark, and keeping the surd to the end is the answer mark.

Method — Two triangles give every exact value you need

center Answer shapes (marked by method marks — M1 for a correct method, A1 for the correct answer): quoting...



45° triangle



30° – 60° triangle

The 45-45-90 and 30-60-90 triangles with their exact side lengths

2

Practice

Practice 2.1 ■ 1 mark

Write down the exact value of $\sin 60^\circ$.

Solution 2.1

Method: Two triangles give every exact value you need — p.4

Worked steps

$$\frac{\sqrt{3}}{2}$$

B1

Practice 2.2 ■ 1 mark

Write down the exact value of $\tan 45^\circ$.

Solution 2.2

Method: Two triangles give every exact value you need — p.4

Worked steps

1

B1

Practice 2.3 ■ 1 mark

Write down the exact value of $\cos 30^\circ$.

Solution 2.3

Method: Two triangles give every exact value you need — p.4

Worked steps

$$\frac{\sqrt{3}}{2}$$

B1

3

Exam-style

Exam-style 3.1 ■ 2 marks

A right-angled triangle has a hypotenuse of 12 cm and an angle of 30° . Find the exact length of the side opposite the 30° angle.

Solution 3.1

Method: Two triangles give every exact value you need — p.4

Worked steps

$$\blacksquare \text{ opposite} = \text{hypotenuse} \times \sin 30^\circ = 12 \times \frac{1}{2} \quad \text{M1}$$

$$= 6 \text{ cm}$$

A1

Exam-style 3.2 ■ 4 marks

Find the exact area of an equilateral triangle of side 8 cm, in the form $a\sqrt{3} \text{ cm}^2$.

Solution 3.2

Method: Two triangles give every exact value you need — p.4

Worked steps

■ split the triangle down the middle to make a 30–60–90 triangle **M1**

■ the height $= 8 \sin 60^\circ = 8 \times \frac{\sqrt{3}}{2} = 4\sqrt{3}$ cm **M1** **A1**

■ area $= \frac{1}{2} \times 8 \times 4\sqrt{3}$

$$= 16\sqrt{3} \text{ cm}^2$$

A1

Exam-style 3.3 ■ 2 marks

Solve $\sin x = \frac{\sqrt{2}}{2}$ for $0^\circ \leq x \leq 180^\circ$.

Solution 3.3

Method: Two triangles give every exact value you need — p.4

Worked steps

■ $\sin 45^\circ = \frac{\sqrt{2}}{2}$, so $x = 45^\circ$ **M1**

■ $\sin$ is also positive in the second quadrant, so $x = 180 - 45$

$$x = 45^\circ \text{ or } 135^\circ$$

A1