

Exercise walk-through

E5.5

Compound shapes and parts of shapes

IGCSE Mathematics

You must be able to

- find an arc length and a sector area using the fraction $\frac{\theta}{360}$
- split a compound shape into rectangles, triangles and parts of circles
- work backwards from a given arc length or sector area to the angle or the radius

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The method

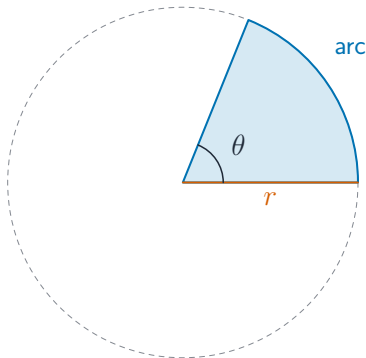
Method — A sector is a fraction of the whole circle

- **Arc length** $= \frac{\theta}{360} \times 2\pi r$. For $r = 10$ and $\theta = 72^\circ$ that is 12.6 cm.
- **Sector area** $= \frac{\theta}{360} \times \pi r^2$. Same fraction, different formula.
- **Compound shapes**: draw the split lines on the diagram and label each piece. A perimeter follows only the OUTSIDE edge, so the internal split lines are not counted.
- **Backwards**: from a sector area, $\theta = \frac{A \times 360}{\pi r^2}$.

Answer shapes (marked by **method marks** — M1 for a correct method, A1 for the correct answer): the fraction $\theta/360$ and the circle formula are separate method marks, so show both.

Method — A sector is a fraction of the whole circle

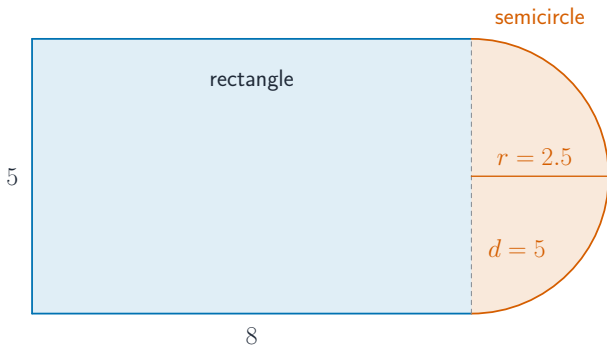
Answer shapes (marked by method marks — M1 for a correct method, A1 for the correct answer): the fraction



A sector of a circle with its angle, arc and radius labelled

Method — A sector is a fraction of the whole circle

Answer shapes (marked by method marks — M1 for a correct method, A1 for the correct answer): the fraction



A compound shape split into simpler pieces

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Practice

Practice 2.1 ■ 3 marks

A sector of a circle has radius 10 cm and angle 72° . Find its arc length, correct to 3 significant figures.

Solution 2.1

Method: A sector is a fraction of the whole circle — p.4

Worked steps

$$\blacksquare \text{ fraction} = \frac{72}{360} = \frac{1}{5} \quad \text{M1}$$

$$\blacksquare \text{ arc} = \frac{1}{5} \times 2 \times \pi \times 10 \quad \text{M1}$$

$$= 12.566 \dots = \mathbf{12.6 \text{ cm}}$$

A1

Practice 2.2 ■ 3 marks

Find the area of the same sector, correct to 3 significant figures.

Solution 2.2

Method: A sector is a fraction of the whole circle — p.4

Worked steps

$$\blacksquare \text{ area} = \frac{1}{5} \times \pi \times 10^2 \quad \text{M1} \quad \text{M1}$$

$$= 62.83 \dots = \mathbf{62.8} \text{ cm}^2$$

A1

Practice 2.3 ■ 3 marks

A shape is made from a rectangle 10 cm by 6 cm with a semicircle of diameter 6 cm joined to one short end. Find the total area, correct to 3 significant figures.

Solution 2.3

Method: A sector is a fraction of the whole circle — p.4

Worked steps

- rectangle = $10 \times 6 = 60 \text{ cm}^2$ (M1)
- semicircle of RADIUS 3: $\frac{1}{2} \times \pi \times 3^2 = 14.137 \dots$ (M1)

$$60 + 14.137 = \mathbf{74.1 \text{ cm}^2} \quad (\text{A1})$$

- the diameter is 6, so the radius is 3 — halving it is the step most often missed

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Exam-style

Exam-style 3.1 ■ 8 marks

A running track is made from a rectangle 84 m long and 70 m wide, with a semicircle of diameter 70 m at each end.

Exam-style 3.1 (a) ■ 4 marks

Find the perimeter of the track, correct to 3 significant figures.

Solution 3.1 (a)

Method: A sector is a fraction of the whole circle — p.4

Worked steps

- the two semicircles make one whole circle of diameter 70 m (M1)
- circle circumference = $\pi \times 70 = 219.91 \dots$ m (M1)
- the two straight sides are 84 m each; the ends are curved, so the 70 m widths are NOT part of the perimeter (M1)

$$2 \times 84 + 219.91 = 387.9 \dots = \mathbf{388 \text{ m}}$$

(A1)

Exam-style 3.1 (b) ■ 4 marks

Find the area enclosed, correct to 3 significant figures.

Solution 3.1 (b)

Worked steps

- rectangle = $84 \times 70 = 5880 \text{ m}^2$ (M1)
- the two semicircles make a circle of radius 35 m (M1)
- circle area = $\pi \times 35^2 = 3848.45 \dots \text{ m}^2$ (M1)

$$5880 + 3848.45 = 9728.4 \dots = \mathbf{9730 \text{ m}^2} \quad (\text{A1})$$

Exam-style 3.2 ■ 3 marks

A sector of a circle of radius 8 cm has an area of 33.5 cm^2 . Find the angle of the sector, correct to the nearest degree.

Solution 3.2

Method: A sector is a fraction of the whole circle — p.4

Worked steps

$$\blacksquare 33.5 = \frac{\theta}{360} \times \pi \times 8^2 \quad \text{M1}$$

$$\blacksquare \theta = \frac{33.5 \times 360}{\pi \times 64} \quad \text{M1}$$

$$= 59.98 \dots = 60^\circ$$

A1