Exercise walk-through

E4.8
Circle theorems II

IGCSE Mathematics

You must be able to

- use the fact that a tangent meets a radius at 90°
- use the fact that two tangents from the same external point are equal
- use the alternate segment theorem, and the perpendicular-from-centre property of a chord

1 The method

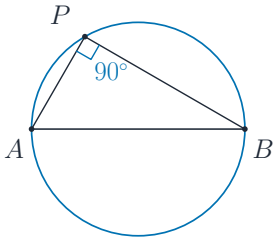
Method — Name the theorem, then do the arithmetic

- **Tangent and radius:** the angle between them at the point of contact is $90°$.
- **Equal tangents:** if TA and TB are tangents from T, then $TA = TB$, and OT bisects angle ATB.
- **Alternate segment:** the angle between a tangent and a chord equals the angle in the alternate segment — the angle "on the other side" of the chord.
- **Chord:** the perpendicular from the centre bisects the chord, which turns a chord question into Pythagoras.

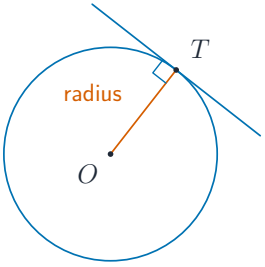
Answer shapes (marked by **method marks** — M1 for a correct method, A1 for the correct answer): the NAME of the theorem is a mark. "Angle $OAT = 90°$ (tangent meets radius)" earns more than the number alone.

Method — Name the theorem, then do the arithmetic

Answer shapes (marked by method marks — M1 for a correct method, A1 for the correct answer): the NAME of the...



angle in a semicircle $= 90°$

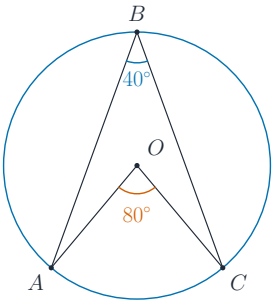


tangent meets radius $= 90°$

A tangent meeting a radius, and the angle in a semicircle

Method — Name the theorem, then do the arithmetic

Answer shapes (marked by method marks — M1 for a correct method, A1 for the correct answer): the NAME of the...



angle at centre $= 2\times$ angle at circumference

The angle at the centre is twice the angle at the circumference

2 Practice

Practice 2.1 ▪ 2 marks

TA and TB are tangents from T to a circle. Given that $TA = 7$ cm, write down the length of TB and name the property used.

Solution 2.1

Method: Name the theorem, then do the arithmetic — p.4

Worked steps

- tangents from an external point are equal in length

$$TB = \mathbf{7}\text{ cm}$$ B1 B1 *for the reason*

Practice 2.2 ▪ 1 mark

Write down the size of the angle between a tangent and the radius drawn to the point of contact.

Solution 2.2

Method: Name the theorem, then do the arithmetic — p.4

Worked steps

- $\mathbf{90^\circ}$ B1

Practice 2.3 ▪ 1 mark

The angle between a tangent at A and a chord AB is $58°$. Write down the angle in the alternate segment.

Solution 2.3

Method: Name the theorem, then do the arithmetic — p.4

Worked steps

- the alternate segment theorem gives the same angle

$$= \mathbf{58}^\circ \qquad \text{B1}$$

3 Exam-style

Exam-style 3.1 ▪ 4 marks

TA and TB are tangents from T to a circle with centre O, and angle $ATB = 46°$.

Exam-style 3.1 (a) ▪ 1 mark

Write down the size of angle OAT, giving a reason.

Solution 3.1 (a)

Method: Name the theorem, then do the arithmetic — p.4

Worked steps

- the tangent meets the radius at the point of contact

$$\mathbf{90^\circ}$$ B1

Exam-style 3.1 (b) ▪ 3 marks

Find the size of angle AOB.

Solution 3.1 (b)

Worked steps

- $OATB$ is a quadrilateral, so its angles add to $360°$ M1
- angles OAT and OBT are both $90°$ M1

$$360 - 90 - 90 - 46 = \mathbf{134}^\circ \quad \text{A1}$$

Exam-style 3.2 ▪ 3 marks

A chord PQ of length 24 cm is drawn in a circle of radius 13 cm. Find the distance from the centre to the chord.

Solution 3.2

Method: Name the theorem, then do the arithmetic — p.4

Worked steps

- the perpendicular from the centre bisects the chord, so half of it is 12 cm **M1**
- Pythagoras with the radius as the hypotenuse: $d = \sqrt{13^2 - 12^2}$ **M1**

$$= \sqrt{25} = \mathbf{5} \text{ cm}$$ **A1**