

Exercise walk-through

E3.7

Perpendicular lines

IGCSE Mathematics

You must be able to

- write down the gradient of a line perpendicular to a given line
- find the equation of the perpendicular through a given point
- find the perpendicular bisector of a line segment

1

The method

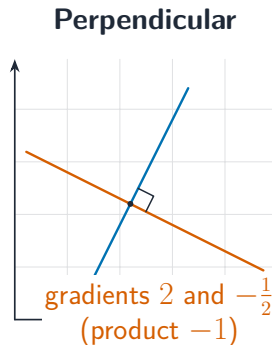
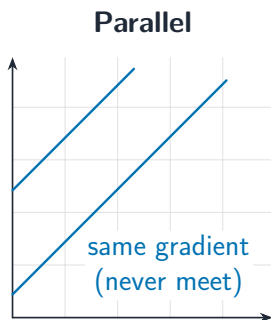
Method — Turn the fraction upside down and change the sign

- **Rule:** $m_1 \times m_2 = -1$, so $m_2 = -\frac{1}{m_1}$ — the **negative reciprocal**.
- **Worked:** perpendicular to a gradient of 4 is $-\frac{1}{4}$; perpendicular to $-\frac{2}{3}$ is $+\frac{3}{2}$. The sign always flips.
- **Through a point:** find the new gradient first, then substitute the point to find c .
Perpendicular to $y = 2x - 1$ through $(4, 3)$: $m = -\frac{1}{2}$, so $3 = -2 + c$ and $y = -\frac{1}{2}x + 5$.
- **Perpendicular bisector:** it passes through the MIDPOINT of the segment and is perpendicular to it — two separate facts, and both are needed.

Answer shapes (marked by **method marks** — M1 for a correct method, A1 for the correct answer): the negative reciprocal, and the substitution for c , are separate method marks.

Method — Turn the fraction upside down and change the sign

Answer shapes (marked by method marks — M1 for a correct method, A1 for the correct answer): the negative...



Parallel and perpendicular lines compared on one grid

2

Practice

Practice 2.1 ■ 1 mark

Write down the gradient of a line perpendicular to $y = 4x + 1$.

Solution 2.1

Method: Turn the fraction upside down and change the sign — p.4

Worked steps

- the negative reciprocal of 4

$$= -\frac{1}{4}$$

B1

Practice 2.2 ■ 1 mark

Write down the gradient of a line perpendicular to $y = -\frac{2}{3}x + 5$.

Solution 2.2

Method: Turn the fraction upside down and change the sign — p.4

Worked steps

- the negative reciprocal of $-\frac{2}{3}$: turn it over and change the sign

$$= \frac{3}{2}$$

B1

Practice 2.3 ■ 3 marks

Find the equation of the line perpendicular to $y = 2x - 1$ that passes through $(4, 3)$.

Solution 2.3

Method: Turn the fraction upside down and change the sign — p.4

Worked steps

- perpendicular gradient $= -\frac{1}{2}$ **M1**
- substitute $(4, 3)$: $3 = -\frac{1}{2}(4) + c = -2 + c$, so $c = 5$ **M1**

$$y = -\frac{1}{2}x + 5$$

A1

3

Exam-style

Exam-style 3.1 ■ 6 marks

A is the point $(1, 2)$ and B is the point $(5, 10)$.

Exam-style 3.1 (a) ■ 2 marks

Find the gradient of AB .

Solution 3.1 (a)

Method: Turn the fraction upside down and change the sign — p.4

Worked steps

$$\blacksquare \quad \frac{10 - 2}{5 - 1} = \frac{8}{4} \quad \text{M1}$$

$$= 2$$

A1

Exam-style 3.1 (b) ■ 4 marks

Find the equation of the perpendicular bisector of AB .

Solution 3.1 (b)

Worked steps

- the midpoint of AB is $\left(\frac{1+5}{2}, \frac{2+10}{2}\right) = (3, 6)$ **M1**
- the perpendicular gradient is $-\frac{1}{2}$ **M1**
- substitute $(3, 6)$: $6 = -\frac{3}{2} + c$, so $c = 7.5$ **M1**

$$y = -\frac{1}{2}x + 7.5 \quad \textbf{A1}$$

Exam-style 3.2 ■ 3 marks

Show that the lines $y = 3x + 2$ and $x + 3y = 9$ are perpendicular.

Solution 3.2

Method: Turn the fraction upside down and change the sign — p.4

Worked steps

- rearrange the second: $3y = -x + 9$, so $y = -\frac{1}{3}x + 3$ **M1**
- the gradients are 3 and $-\frac{1}{3}$ **M1**

$$3 \times \left(-\frac{1}{3}\right) = -1, \text{ so the lines are perpendicular}$$

A1