

Exercise walk-through

E3.4

Length and midpoint

IGCSE Mathematics

You must be able to

- find the length of a line segment from the coordinates of its ends
- find the midpoint of a line segment
- find one endpoint when the midpoint and the other endpoint are known

1

The method

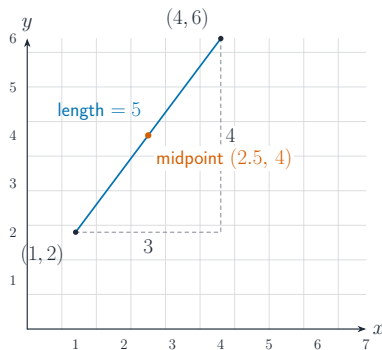
Method — Length is Pythagoras; midpoint is an average

- **Length:** $AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$. Both differences are SQUARED, so their signs do not matter.
- **Midpoint:** $M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$ — the average of the x 's and the average of the y 's.
- **Worked:** for $A(1, 2)$ and $B(7, 10)$, $AB = \sqrt{6^2 + 8^2} = \sqrt{100} = 10$ and $M = (4, 6)$.
- **Backwards:** if M is the midpoint of PQ , then $Q = (2x_M - x_P, 2y_M - y_P)$.

Answer shapes (marked by **method marks** — M1 for a correct method, A1 for the correct answer): the line showing the squares being added, before the root is taken, is the method mark.

Method — Length is Pythagoras; midpoint is an average

Answer shapes (marked by method marks — M1 for a correct method, A1 for the correct answer): the line showing...



A line segment on a grid with its length and midpoint marked

2

Practice

Practice 2.1 ■ 2 marks

A is $(1, 2)$ and B is $(7, 10)$. Find the length of AB .

Solution 2.1

Worked steps

■ differences are 6 across and 8 up, so $AB = \sqrt{6^2 + 8^2} = \sqrt{36 + 64}$ **M1**

$$= \sqrt{100} = 10 \quad \text{A1}$$

Practice 2.2 ■ 2 marks

Find the midpoint of AB for the same two points.

Solution 2.2

Worked steps

■ average each coordinate: $\left(\frac{1+7}{2}, \frac{2+10}{2}\right)$ **M1**

$$= (4, 6)$$

A1

Practice 2.3 ■ 2 marks

P is $(-3, 5)$ and Q is $(2, -7)$. Find the length of PQ .

Solution 2.3

Worked steps

■ differences are 5 and -12 , so $PQ = \sqrt{5^2 + (-12)^2} = \sqrt{25 + 144}$ **M1**

$$= \sqrt{169} = \mathbf{13} \quad \mathbf{A1}$$

3

Exam-style

Exam-style 3.1 ■ 5 marks

A is the point $(-2, 1)$ and B is the point $(4, 6)$.

Exam-style 3.1 (a) ■ 2 marks

Find the coordinates of the midpoint of AB .

Solution 3.1 (a)

Method: Length is Pythagoras; midpoint is an average — p.4

Worked steps

$$\blacksquare \left(\frac{-2 + 4}{2}, \frac{1 + 6}{2} \right) \quad \text{M1}$$

$$= (1, 3.5)$$

A1

Exam-style 3.1 (b) ■ 3 marks

Find the length of AB , giving your answer in exact form.

Solution 3.1 (b)

Worked steps

- differences are 6 and 5 (M1)

- $AB = \sqrt{36 + 25}$ (M1)

$$= \sqrt{61}$$

(A1)

- 61 has no square factor, so $\sqrt{61}$ is already the exact form; a decimal would lose the mark

Exam-style 3.2 ■ 3 marks

$M(3, -1)$ is the midpoint of PQ , and P is the point $(-1, 4)$. Find the coordinates of Q .

Solution 3.2

Worked steps

- the midpoint is the average, so $\frac{-1 + x_Q}{2} = 3$ and $\frac{4 + y_Q}{2} = -1$ **M1**
- $x_Q = 6 - (-1) = 7$ and $y_Q = -2 - 4 = -6$ **M1**

$$Q = (7, -6)$$

A1