Exercise walk-through

Coordinates ▪ E3.1

eXercise

You must be able to

- plot and read coordinates (x, y)
- find the length of a segment using Pythagoras
- find the midpoint of a segment

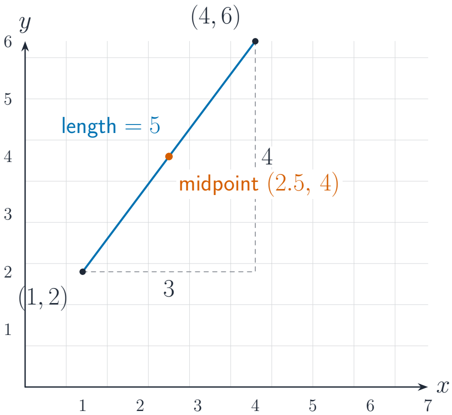
Method — Length and midpoint

$$\text{length} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \qquad \text{midpoint} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

- **Worked** $(1, 2)$ **to** $(4, 6)$**:** length $= \sqrt{3^2 + 4^2} = 5$; midpoint $= (2.5, 4)$.

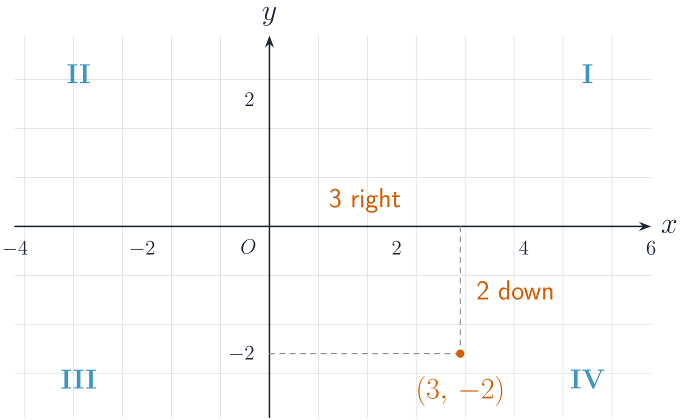
Answer shapes: M1 a correct method, **A1** the correct answer.

Method — Length and midpoint



A right triangle gives the length; the midpoint is the average of the coordinates

Method — Length and midpoint



Points are plotted on the coordinate plane

Method — The three coordinate formulas, and when each is wanted

For $A(x_1, y_1)$ and $B(x_2, y_2)$:

$$\text{midpoint} = \left(\frac{x_1 + x_2}{2}, \ \frac{y_1 + y_2}{2}\right) \qquad \text{length} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \qquad \text{gradient} = \frac{y_2 -}{x_2 -}$$

The midpoint **adds** and halves; the length and gradient **subtract**. Mixing the two is the commonest error, and it is easy to spot: a midpoint should lie between the two points.

Parallel and perpendicular. Parallel lines have the **same** gradient. Perpendicular lines have gradients whose product is -1, so the perpendicular gradient is the **negative reciprocal**: if $m = \frac{2}{3}$ then the perpendicular gradient is $-\frac{3}{2}$.

The equation of a line is easiest from $y - y_1 = m(x - x_1)$ using any point on it; rearrange to $y = mx + c$ at the end if the question asks for that form.

Method — The three coordinate formulas, and when each is wanted

Worked example. $A(1,1)$ and $B(7,9)$. Find the equation of the perpendicular bisector of AB.

1. Midpoint: $\left(\frac{1+7}{2}, \frac{1+9}{2}\right) = (4,5)$.
2. Gradient of AB: $\dfrac{9-1}{7-1} = \dfrac{8}{6} = \dfrac{4}{3}$.
3. Perpendicular gradient: the negative reciprocal, $-\dfrac{3}{4}$.
4. Through the midpoint: $y - 5 = -\frac{3}{4}(x-4)$.
5. Rearranged: $y = -\frac{3}{4}x + 8$.

The bisector must pass through the **midpoint** — using A or B instead gives a parallel line and loses every mark after step 3.

Practice 2.1 ▪ 2 marks

Find the midpoint of the segment joining $(2, 3)$ and $(6, 9)$.

Solution 2.1

Method: The three coordinate formulas, and when each is wanted

Worked steps

$\left(\frac{2+6}{2}, \frac{3+9}{2}\right) = (4, 6)$ **M1** **A1** *for the midpoint method; for* $(4, 6)$.

Practice 2.2 ▪ 2 marks

Find the length of the segment from $(0, 0)$ to $(3, 4)$.

Solution 2.2

Method: The three coordinate formulas, and when each is wanted

Worked steps

$\sqrt{3^2+4^2} = \sqrt{25} = 5$ **M1** **A1** *for* $\sqrt{9+16}$; *for* 5.

Practice 2.3 ▪ 1 mark

Write down the coordinates of the point 3 left and 2 up from the origin.

Solution 2.3

Method: The three coordinate formulas, and when each is wanted

Worked steps

$$\frac{7-(-1)}{6-2} = \frac{8}{4} \quad \textbf{M1} \quad = 2 \quad \textbf{A1}.$$

Practice 2.3 ▪ 2 marks

Find the gradient of the line joining $(2, -1)$ and $(6, 7)$.

Solution 2.3

Method: The three coordinate formulas, and when each is wanted

Worked steps

$$\frac{7-(-1)}{6-2} = \frac{8}{4} \;\; \textbf{M1} = 2 \;\; \textbf{A1}.$$

Practice 2.4 ▪ 1 mark

State the gradient of a line perpendicular to $y = \frac{2}{5}x + 1$.

Solution 2.4

Method: The three coordinate formulas, and when each is wanted

Worked steps

$-\dfrac{5}{2}$ **B1**.

Practice 2.5 ▪ 2 marks

Find the length of the line joining $(0,0)$ and $(9,12)$.

Solution 2.5

Method: The three coordinate formulas, and when each is wanted

Worked steps

$\sqrt{9^2 + 12^2} = \sqrt{81 + 144} = \sqrt{225}$ **M1** $= 15$ **A1**.

Exam-style 3.1 ▪ 2 marks

$A = (1, 1)$ and $B = (7, 9)$.

(a) Find the midpoint of AB.

(b) Find the length of AB.

Solution 3.1

Method: The three coordinate formulas, and when each is wanted

Worked steps

(a) $\left(\frac{1+7}{2}, \frac{1+9}{2}\right) = (4, 5)$ M1 A1. (b) $\sqrt{(7-1)^2 + (9-1)^2} = \sqrt{36+64} = \sqrt{100} = 10$ M1 A1 *for* 10.

Exam-style 3.2 ▪ 2 marks

$M(3,5)$ is the midpoint of PQ. Given $P=(1,2)$, find the coordinates of Q.

Solution 3.2

Worked steps

(a) $\left(\dfrac{-2+6}{2}, \dfrac{3+9}{2}\right)$ M1 $= (2,6)$ A1.

(b) $\sqrt{(6-(-2))^2+(9-3)^2} = \sqrt{8^2+6^2}$ M1 $= \sqrt{100}$ M1 $= 10$ A1.

(c) $\dfrac{9-3}{6-(-2)} = \dfrac{6}{8}$ M1 $= \dfrac{3}{4}$ A1.

(d) $y-3 = \frac{3}{4}(x+2)$ M1; $y = \frac{3}{4}x + \frac{3}{2} + 3$ M1; $y = \frac{3}{4}x + 4.5$ A1.

(e) Perpendicular gradient $= -\dfrac{4}{3}$ M1; through the midpoint $(2,6)$:

$y-6 = -\frac{4}{3}(x-2)$ M1; $y = -\frac{4}{3}x + \frac{26}{3}$ A1.

Exam-style 3.2 ▪ 2 marks

$P = (-2, 3)$ and $Q = (6, 9)$.

(a) Find the midpoint of PQ.

(b) Find the length of PQ.

(c) Find the gradient of PQ.

(d) Find the equation of the line through P and Q, giving your answer in the form $y = mx + c$.

(e) Find the equation of the perpendicular bisector of PQ.

Solution 3.2

Method: The three coordinate formulas, and when each is wanted

Worked steps

(a) $\left(\dfrac{-2+6}{2}, \dfrac{3+9}{2}\right)$ **M1** $= (2,6)$ **A1**.

(b) $\sqrt{(6-(-2))^2+(9-3)^2} = \sqrt{8^2+6^2}$ **M1** $= \sqrt{100}$ **M1** $= 10$ **A1**.

(c) $\dfrac{9-3}{6-(-2)} = \dfrac{6}{8}$ **M1** $= \dfrac{3}{4}$ **A1**.

(d) $y-3 = \frac{3}{4}(x+2)$ **M1**; $y = \frac{3}{4}x + \frac{3}{2} + 3$ **M1**; $y = \frac{3}{4}x + 4.5$ **A1**.

(e) Perpendicular gradient $= -\dfrac{4}{3}$ **M1**; through the midpoint $(2,6)$:
$y-6 = -\frac{4}{3}(x-2)$ **M1**; $y = -\frac{4}{3}x + \frac{26}{3}$ **A1**.