

Exercise walk-through

E2.7

Sequences

IGCSE Mathematics

You must be able to

- find the n th term of a linear sequence from its common difference
- recognise a quadratic sequence from its constant second difference and find its n th term
- recognise a cubic or an exponential sequence and write down its n th term

1

The method

Method — The differences tell you which kind of sequence it is

- **Linear:** constant FIRST difference d , so the n th term is $dn + c$. For 5, 8, 11, 14, $d = 3$ and the n th term is $3n + 2$.
- **Quadratic:** constant SECOND difference. Half of it is the coefficient of n^2 , then subtract an^2 from each term and the remainder is linear. For 3, 8, 15, 24 the second difference is 2, so the n th term is $n^2 + 2n$.
- **Cubic:** the THIRD difference is constant. 1, 8, 27, 64 is n^3 .
- **Exponential:** each term is the previous one $\times r$. For 2, 6, 18, 54 the n th term is $2 \times 3^{n-1}$.

Answer shapes (marked by **method marks** — M1 for a correct method, A1 for the correct answer): writing the difference row down is the method mark, and it also tells you which method to use.

Method — The differences tell you which kind of sequence it is

Answer shapes (marked by method marks — M1 for a correct method, A1 for the correct answer): writing the...



A sequence of patterns growing by a fixed amount each time

2

Practice

Practice 2.1 ■ 2 marks

Find the n th term of the sequence 7, 12, 17, 22, ...

Solution 2.1

Method: The differences tell you which kind of sequence it is — p.4

Worked steps

- the common difference is 5, so the n th term starts $5n$; at $n = 1$, $5 \times 1 = 5$ and the term is 7 **M1**

$$= 5n + 2$$

A1

Practice 2.2 ■ 2 marks

Write down the next two terms of the sequence 4, 12, 36, 108, ...

Solution 2.2

Method: The differences tell you which kind of sequence it is — p.4

Worked steps

■ each term is $\times 3$ **M1**

324 and 972

A1

Practice 2.3 ■ 2 marks

Find the n th term of the sequence 3, 12, 48, 192, ...

Solution 2.3

Method: The differences tell you which kind of sequence it is — p.4

Worked steps

- each term is $\times 4$ and the first term is 3 M1

$$= 3 \times 4^{n-1}$$

A1

3

Exam-style

Exam-style 3.1 ■ 4 marks

The first five terms of a sequence are 2, 9, 20, 35, 54.

Exam-style 3.1 (a) ■ 1 mark

Write down the next term.

Solution 3.1 (a)

Method: The differences tell you which kind of sequence it is — p.4

Worked steps

- first differences 7, 11, 15, 19 rise by 4, so the next is 23

$$54 + 23 = \mathbf{77}$$

B1

Exam-style 3.1 (b) ■ 3 marks

Find an expression for the n th term.

Solution 3.1 (b)

Worked steps

- the second difference is 4, so the n^2 coefficient is $4 \div 2 = 2$ **M1**
- subtract $2n^2$, i.e. 2, 8, 18, 32, 50, from the terms: 0, 1, 2, 3, 4 **M1**
- that remainder is $n - 1$

$$= 2n^2 + n - 1$$

A1

Exam-style 3.2 ■ 1 mark

Write down the n th term of the sequence 1, 8, 27, 64, ...

Solution 3.2

Method: The differences tell you which kind of sequence it is — p.4

Worked steps

- the terms are $1^3, 2^3, 3^3, 4^3$

$$= n^3$$

B1