Exercise walk-through

Inequalities ▪ E2.6

eXercise

You must be able to

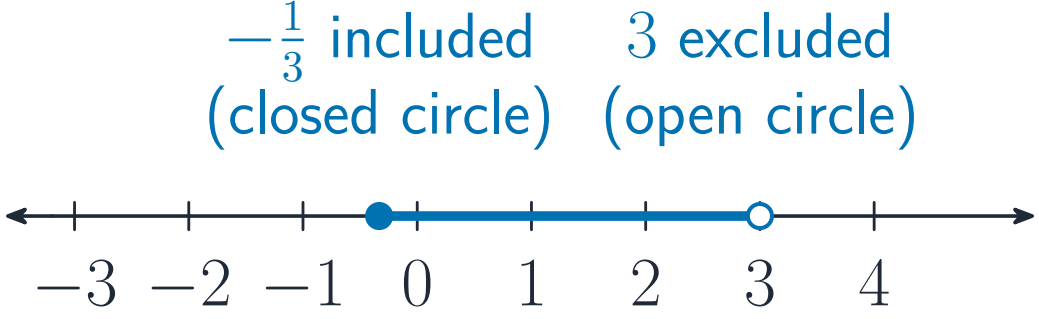
- solve a linear inequality (and reverse the sign when needed)
- solve a double inequality
- list the integers that satisfy an inequality

Method — Solving inequalities

- **Worked:** $-3 \leqslant 3x - 2 < 7 \to$ add 2: $-1 \leqslant 3x < 9 \to \div 3$: $-\frac{1}{3} \leqslant x < 3$.
- **Reverse:** $-2x < 6 \to x > -3$ (sign flips when dividing by -2).
- **Number line:** closed circle for $\leqslant$ / $\geqslant$, open circle for $<$ / $>$.

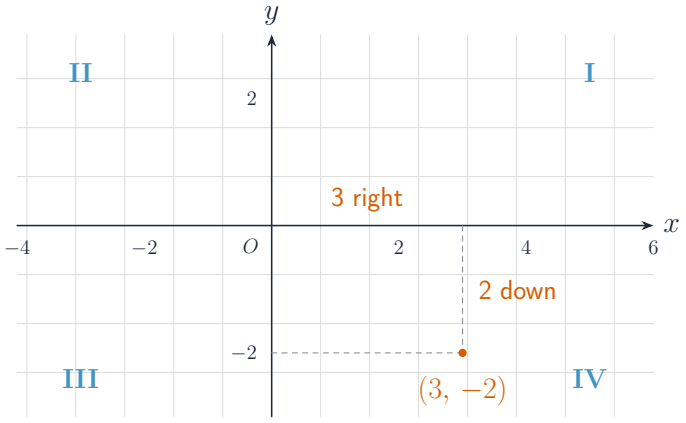
Answer shapes: **M1** a correct method, **A1** the correct answer.

Method — Solving inequalities



A number line: closed circle includes the value, open circle excludes it

Method — Solving inequalities



Two-variable inequalities shade regions of the plane

Method — Inequalities: the same steps as an equation, with one extra rule

Solve an inequality exactly as you would the matching equation — **except** that multiplying or dividing both sides by a **negative** number **reverses** the inequality sign.

$$-3x > 12 \;\Rightarrow\; x < -4$$

That single rule is the difference between full marks and none on half the questions in this topic. If you would rather not risk it, move the x term to the side where it is positive instead: $-3x > 12$ becomes $-12 > 3x$, so $-4 > x$, which is the same answer with no sign flip.

A double inequality is solved by doing the same operation to all three parts: $5 \leqslant 2x + 1 \leqslant 11$ gives $4 \leqslant 2x \leqslant 10$, then $2 \leqslant x \leqslant 5$.

Method — Inequalities: the same steps as an equation, with one extra rule

Reading the answer as integers. For $-2 < x \leqslant 3$: the $<$ EXCLUDES -2 and the $\leqslant$ INCLUDES 3, so the integers are $-1, 0, 1, 2, 3$. Getting the endpoints right is usually a mark by itself.

On a number line, an open circle means the value is excluded ($<$ or $>$) and a filled circle means it is included ($\leqslant$ or $\geqslant$).

Worked example. Find the integers n for which $-9 < 3n - 3 \leqslant 6$.

1. Add 3 throughout: $-6 < 3n \leqslant 9$.
2. Divide by 3 throughout: $-2 < n \leqslant 3$.
3. The endpoints: -2 is excluded, 3 is included.
4. So $n = -1, 0, 1, 2, 3$.

Practice 2.1 ▪ 2 marks

Solve $2x + 1 > 9$.

Solution 2.1

Worked steps

$2x > 8 \to x > 4$ M1 A1 *for* $2x > 8$*; for* $x > 4$.

Practice 2.2 ▪ 2 marks

Solve $-3x \leqslant 12$.

Solution 2.2

Worked steps

dividing by -3 reverses the sign: $x \geqslant -4$ M1 A1 *for dividing by* -3 *and reversing; for* $x \geqslant -4$.

Practice 2.3 ▪ 2 marks

Solve $5x - 4 < 11$.

Solution 2.3

Worked steps

$5x < 15 \to x < 3$ M1 A1 *for* $x < 3$.

Practice 2.4 ▪ 2 marks

Solve $-4x + 3 > 19$.

Solution 2.4

Worked steps

$-4x > 16$ **M1**; dividing by -4 reverses the sign, so $x < -4$ **A1**.

Practice 2.5 ▪ 2 marks

Write down the inequality shown by an open circle at -1 and a filled circle at 4, joined by a line.

Solution 2.5

Method: Inequalities: the same steps as an equation, with one extra rule

Worked steps

$-1 < x \leqslant 4$ **M1** **A1** *for the correct endpoints, for the correct strictness at each.*

Exam-style 3.1 ▪ 3 marks

Solve the inequality $5 \leqslant 2x + 1 \leqslant 11$.

Solution 3.1

Worked steps

subtract 1: $4 \leqslant 2x \leqslant 10$; divide by 2: $2 \leqslant x \leqslant 5$ **M1** **M1** **A1** *for subtracting* 1*; for dividing by* 2*; for* $2 \leqslant x \leqslant 5$.

Exam-style 3.2 ▪ 2 marks

Write down the integers that satisfy $-2 < x \leqslant 3$.

Solution 3.2

Method: Inequalities: the same steps as an equation, with one extra rule

Worked steps

the integers are $-1, 0, 1, 2, 3$ M1 A1 *for excluding* -2 *but including* 3*; for the full list.*

Exam-style 3.3 ▪ 2 marks

Solve $7 - 2x \geqslant 1$.

Solution 3.3

Worked steps

$-2x \geqslant -6 \rightarrow$ dividing by -2 reverses the sign $\rightarrow x \leqslant 3$ **M1** **A1** *for* $-2x \geqslant -6$ *and reversing; for* $x \leqslant 3$.

Exam-style 3.4 ▪ 3 marks

(a) Solve the inequality $-11 < 4x + 5 \leqslant 13$.
(b) Write down all the integers that satisfy your answer to part (a).
(c) Solve $9 - 4x \geqslant 21$, and explain the step in which the inequality sign changes.
(d) A rectangle has width x cm and length $(x + 5)$ cm. Its perimeter is at least 30 cm and less than 50 cm. Form an inequality in x and solve it.

Solution 3.4

Method: Inequalities: the same steps as an equation, with one extra rule

Worked steps

(a) Subtract 5 throughout: $-16 < 4x \leqslant 8$ M1; divide by 4 throughout: $-4 < x \leqslant 2$ M1 A1.

(b) $-3, -2, -1, 0, 1, 2$ M1 A1 *for excluding -4 and including 2; for the full list.*

(c) $-4x \geqslant 12$ M1; dividing both sides by -4 reverses the inequality M1; $x \leqslant -3$ A1.

(d) Perimeter $= 2(x) + 2(x + 5) = 4x + 10$ M1; $30 \leqslant 4x + 10 < 50$ M1; $20 \leqslant 4x < 40$ M1; $5 \leqslant x < 10$ A1.