

Exercise walk-through

E2.3

Algebraic fractions

IGCSE Mathematics

You must be able to

- simplify an algebraic fraction by factorising the numerator and the denominator
- add or subtract algebraic fractions, writing the result as a single fraction
- cancel only a whole factor, never a single term

1

The method

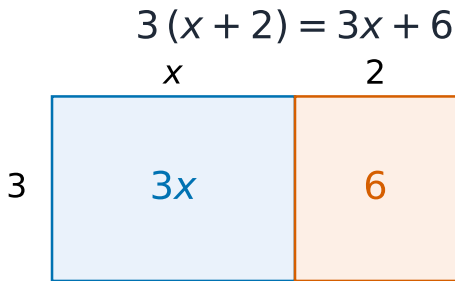
Method — Factorise first — only a whole bracket can cancel

- **Simplify:** $\frac{x^2 - 9}{x^2 + 7x + 12} = \frac{(x-3)(x+3)}{(x+3)(x+4)} = \frac{x-3}{x+4}$ — the bracket $(x+3)$ cancels because it is a **factor** of both.
- **Never cancel a term.** In $\frac{x+3}{x+4}$ the x 's do **not** cancel; they are added, not multiplied.
- **Adding:** use the product of the denominators. $\frac{3x}{4} + \frac{x}{6} = \frac{9x + 2x}{12} = \frac{11x}{12}$.
- **Subtracting:** put the whole second numerator in a bracket, then subtract — $-3(x+1)$ is $-3x - 3$, not $-3x + 3$.

Answer shapes (marked by **method marks** — M1 for a correct method, A1 for the correct answer): each correct factorisation, and the single combined fraction before simplifying, earns a method mark.

Method — Factorise first — only a whole bracket can cancel

Answer shapes (marked by method marks — M1 for a correct method, A1 for the correct answer): each correct...



$a(b + c) = ab + ac$ · area model of expansion

An area model showing a quadratic split into its factors

2

Practice

Practice 2.1 ■ 3 marks

Simplify $\frac{x^2 - 25}{x^2 - 2x - 15}$.

Solution 2.1

Method: Factorise first — only a whole bracket can cancel — p.4

Worked steps

- numerator is a difference of two squares: $x^2 - 25 = (x - 5)(x + 5)$ **M1**
- denominator factorises: $x^2 - 2x - 15 = (x - 5)(x + 3)$ **M1**

$$= \frac{x + 5}{x + 3} \quad \text{A1}$$

Practice 2.2 ■ 2 marks

Write $\frac{x}{3} + \frac{2x}{5}$ as a single fraction.

Solution 2.2

Method: Factorise first — only a whole bracket can cancel — p.4

Worked steps

■ common denominator 15: $\frac{5x}{15} + \frac{6x}{15}$ **M1**

$$= \frac{11x}{15} \quad \textbf{A1}$$

Practice 2.3 ■ 2 marks

Simplify $\frac{6x^2y}{9xy^3}$.

Solution 2.3

Method: Factorise first — only a whole bracket can cancel — p.4

Worked steps

- divide the numbers by 3 and cancel one x and one y **M1**

$$= \frac{2x}{3y^2}$$

A1

3

Exam-style

Exam-style 3.1 ■ 4 marks

Write $\frac{3}{x-2} + \frac{2}{x+1}$ as a single fraction in its simplest form.

Solution 3.1

Method: Factorise first — only a whole bracket can cancel — p.4

Worked steps

■ common denominator $(x-2)(x+1)$ **M1**

■ $\frac{3(x+1) + 2(x-2)}{(x-2)(x+1)}$ **M1**

■ expand the numerator: $3x + 3 + 2x - 4$ **M1**

$$= \frac{5x - 1}{(x-2)(x+1)}$$

A1

Exam-style 3.2 ■ 4 marks

Simplify $\frac{2x^2 + 5x - 3}{4x^2 - 1}$.

Solution 3.2

Method: Factorise first — only a whole bracket can cancel — p.4

Worked steps

■ numerator: $2x^2 + 5x - 3 = (2x - 1)(x + 3)$ M1 M1

■ denominator is a difference of two squares: $4x^2 - 1 = (2x - 1)(2x + 1)$ M1

$$= \frac{x + 3}{2x + 1} \quad \text{A1}$$