Exercise walk-through

Algebraic manipulation ▪ E2.2

eXercise

You must be able to

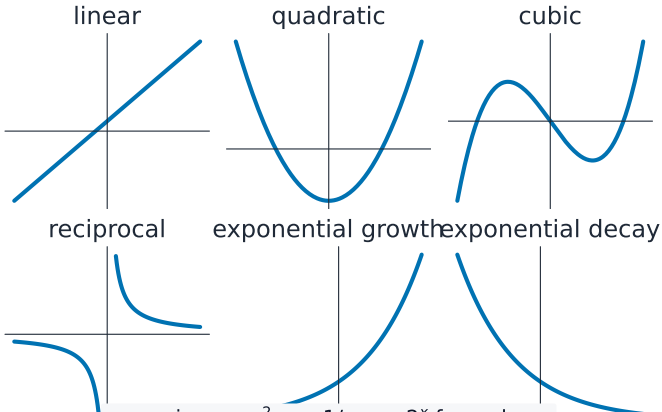
- expand single and double brackets
- factorise by taking out a common factor
- factorise a difference of two squares and a quadratic

Method — Expand and factorise

- **Expand:** $(2x+1)(x-4) = 2x^2 - 7x - 4$.
- **Common factor:** $9x^2 + 15xy = 3x(3x+5y)$.
- **Difference of two squares 平方差:** $9x^2 - 16 = (3x+4)(3x-4)$.
- **Quadratic:** $2x^2 + 7x + 3 = (x+3)(2x+1)$ (split the middle term).

Answer shapes: **M1** a correct method, **A1** the correct answer.

Method — Expand and factorise

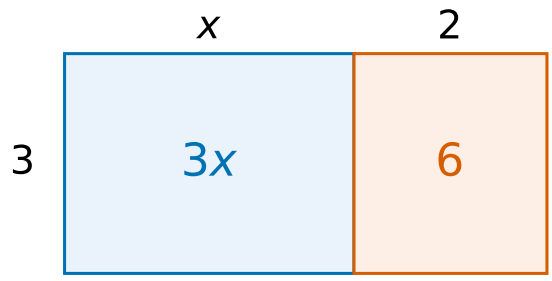


recognise $y = x^2$, $y = 1/x$, $y = 2^x$ from shape

A suspension bridge cable forms a parabola

Method — Expand and factorise

$$3\,(x+2) = 3x + 6$$



$a(b+c) = ab + ac$ · area model of expansion

A rectangle of height 3 splits into areas 3x and 6, so 3(x+2)=3x+6

Method — Choosing the factorising method

Work through these in order; the first one that applies is the right one.

1. **Common factor first, every time.** $6x^2 + 9x = 3x(2x + 3)$. Missing it makes everything afterwards harder, and the mark for the common factor is lost even if the rest is right.
2. **Difference of two squares**, when there are two terms and both are squares with a minus between: $a^2 - b^2 = (a + b)(a - b)$. So $x^2 - 49 = (x + 7)(x - 7)$ and $9x^2 - 25 = (3x + 5)(3x - 5)$.
3. **A quadratic with $a = 1$:** find two numbers that multiply to the constant and add to the coefficient of x. For $x^2 + 3x - 10$ they are $+5$ and -2, so it factorises as $(x + 5)(x - 2)$.
4. **A quadratic with $a \neq 1$:** multiply a by c, find two numbers multiplying to that and adding to b, split the middle term and factorise in pairs.

Method — Choosing the factorising method

5. **Four terms:** factorise in pairs, then take out the common bracket: $xy + 2x + 3y + 6 = x(y+2) + 3(y+2) = (x+3)(y+2)$.

Worked example (the $a \neq 1$ method). Factorise $2x^2 + 5x - 3$.

1. $a \times c = 2 \times (-3) = -6$.
2. Two numbers multiplying to -6 and adding to $+5$: $+6$ and -1.
3. Split the middle term: $2x^2 + 6x - x - 3$.
4. Factorise in pairs: $2x(x+3) - 1(x+3)$.
5. Take out the common bracket: $(2x-1)(x+3)$.

Always **check by expanding**. It takes one line and catches a sign error, which is the commonest loss on this topic.

Practice 2.1 ▪ 1 mark

Expand $3x(2x - 5)$.

Solution 2.1

Worked steps

$3x(2x-5) = 6x^2 - 15x$ B1.

Practice 2.2 ▪ 1 mark

Factorise $6a + 9ab$.

Solution 2.2

Worked steps

$6a + 9ab = 3a(2 + 3b)$ B1.

Practice 2.3 ▪ 2 marks

Expand and simplify $(x+2)(x-5)$.

Solution 2.3

Worked steps

$(x+2)(x-5) = x^2 - 5x + 2x - 10 = x^2 - 3x - 10$ M1 A1 *for three or four correct terms; for* $x^2 - 3x - 10$.

Practice 2.4 ▪ 2 marks

Factorise fully $12x^2 - 27$.

Solution 2.4

Worked steps

Common factor 3: $3(4x^2 - 9)$ M1; difference of two squares: $3(2x + 3)(2x - 3)$ A1.

Practice 2.5 ▪ 2 marks

Factorise $ab + 4a + 3b + 12$.

Solution 2.5

Worked steps

$a(b+4)+3(b+4)$ M1; $(a+3)(b+4)$ A1.

Exam-style 3.1 ▪ 1 mark

Factorise $x^2 - 49$.

Solution 3.1

Worked steps

$x^2 - 49 = (x+7)(x-7)$ B1.

Exam-style 3.2 ▪ 2 marks

Factorise $x^2 + 3x - 10$.

Solution 3.2

Worked steps

two numbers multiplying to -10 and adding to 3 are 5 and -2: $(x+5)(x-2)$ M1 A1 *for the numbers.*

Exam-style 3.3 ▪ 2 marks

Factorise $2x^2 + 5x - 3$.

Solution 3.3

Worked steps

$a \times c = -6$, middle $+5$: numbers 6 and $-1 \to 2x^2+6x-x-3 = 2x(x+3)-(x+3) = (2x-1)(x+3)$ **M1** **A1** *for splitting the middle term; for* $(2x-1)(x+3)$.

Exam-style 3.4 ▪ 3 marks

(a) Factorise fully $18x^2 - 8$.

(b) Factorise $3x^2 - 11x + 6$.

(c) Simplify $\dfrac{x^2 - 9}{x^2 + x - 12}$, showing your factorising.

(d) Hence solve $\dfrac{x^2 - 9}{x^2 + x - 12} = 2$.

Solution 3.4

Worked steps

(a) Common factor 2: $2(9x^2 - 4)$ **M1**; this is a difference of two squares **M1**; $2(3x+2)(3x-2)$ **A1**.

(b) $a \times c = 18$; two numbers multiplying to 18 and adding to -11 are -9 and -2 **M1**; $3x^2 - 9x - 2x + 6 = 3x(x-3) - 2(x-3)$ **M1**; $(3x-2)(x-3)$ **A1**.

(c) Numerator $= (x+3)(x-3)$ **M1**; denominator $= (x+4)(x-3)$ **M1**; cancelling the common $(x-3)$ gives $\dfrac{x+3}{x+4}$ **A1**.

(d) $\dfrac{x+3}{x+4} = 2$, so $x+3 = 2(x+4)$ **M1**; $x+3 = 2x+8$, so $-5 = x$ **M1**; $x = -5$ **A1**.