Exercise walk-through

E2.13
Functions

IGCSE Mathematics

You must be able to

- evaluate $f(a)$ and form composite functions such as $fg(x)$ and $gf(x)$
- find an inverse function by rearranging $y = f(x)$ to make x the subject
- state which values of x must be excluded from a domain

1 The method

Method — $fg(x)$ means "do g first"

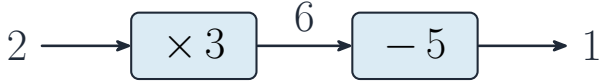
- **Evaluate:** if $f(x) = 3x - 1$ then $f(4) = 11$.
- **Composite:** $fg(x) = f\big(g(x)\big)$ — substitute g INTO f. With $f(x) = 2x$ and $g(x) = x + 3$, $fg(x) = 2(x + 3) = 2x + 6$, while $gf(x) = 2x + 3$. The two are **different**.
- **Inverse:** write $y = f(x)$, rearrange for x, then swap the letters. For $f(x) = 5x + 2$, $y = 5x + 2$ gives $x = \dfrac{y - 2}{5}$, so $f^{-1}(x) = \dfrac{x - 2}{5}$.
- **Domain:** exclude any x that makes a denominator zero or a square root negative.

Answer shapes (marked by **method marks** — M1 for a correct method, A1 for the correct answer): the substitution line, or the rearranged equation before the letters are swapped, is the method mark.

Method — $fg(x)$ means "do g first"

Answer shapes (marked by method marks — M1 for a correct method, A1 for the correct answer): the substitution...

$f(x) = 3x - 5$: do $\times 3$, then -5



$f^{-1}(x) = \dfrac{x+5}{3}$: **reverse the order, undo each step**

the inverse takes the output back to the input: $f(2) = 1$, so $f^{-1}(1) = 2$

A function machine turning an input into an output

2 Practice

Practice 2.1 ▪ 1 mark

$f(x) = 5x + 2$. Find $f(3)$.

Solution 2.1

Worked steps

- $5(3) + 2$

$$= \mathbf{17}$$ B1

Practice 2.2 ▪ 2 marks

$f(x) = 5x + 2$. Find $f^{-1}(x)$.

Solution 2.2

Worked steps

- $y = 5x + 2$, so $x = \dfrac{y-2}{5}$ M1

$$f^{-1}(x) = \frac{\mathbf{x-2}}{\mathbf{5}}$$ A1

Practice 2.3 ▪ 2 marks

$f(x) = 2x$ and $g(x) = x + 3$. Find $fg(x)$ in its simplest form.

Solution 2.3

Worked steps

- do g first, then f: $f(x+3) = 2(x+3)$ M1

$$= \mathbf{2x + 6}$$ A1

3 Exam-style

Exam-style 3.1 ▪ 6 marks

$f(x) = 3x - 4$ and $g(x) = x^2$.

Exam-style 3.1 (a) ▪ 2 marks

Find $gf(2)$.

Solution 3.1 (a)

Worked steps

- $f(2) = 3(2) - 4 = 2$ M1
- then $g(2) = 2^2$

$$= \mathbf{4}$$ A1

Exam-style 3.1 (b) ▪ 2 marks

Find $fg(x)$.

Solution 3.1 (b)

Worked steps

- do g first: $f\left(x^2\right) = 3x^2 - 4$ M1

$$= \mathbf{3x^2 - 4}$$ A1

- note $gf(x) = (3x - 4)^2$, which is not the same

Exam-style 3.1 (c) ▪ 2 marks

Find $f^{-1}(x)$.

Solution 3.1 (c)

Worked steps

- $y = 3x - 4$, so $x = \dfrac{y+4}{3}$ M1

$$f^{-1}(x) = \frac{\mathbf{x+4}}{\mathbf{3}}$$ A1

Exam-style 3.2 ▪ 1 mark

$h(x) = \dfrac{2}{x-1}$. Write down the value of x that must be excluded from the domain of h.

Solution 3.2

Method: \$fg(x)\$ means “do \$g\$ first” — p.4

Worked steps

- the denominator $x - 1$ must not be zero

$$x = \mathbf{1} \qquad \text{B1}$$