Exercise walk-through

E2.12

Differentiation

IGCSE Mathematics

You must be able to

- differentiate a sum of terms of the form ax^n
- find the gradient of a curve at a given point
- find the turning points by solving $\frac{dy}{dx} = 0$, and determine their nature

1 The method

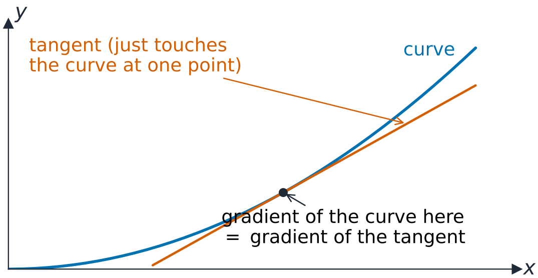
Method — Multiply by the power, then drop the power by one

- **The rule:** if $y = ax^n$ then $\dfrac{dy}{dx} = anx^{n-1}$.
- **A constant differentiates to zero**, and ax differentiates to a — its graph is a straight line of gradient a.
- **Gradient at a point:** differentiate, then substitute the x-value. For $y = x^3 - 4x$ at $x = 2$, $\dfrac{dy}{dx} = 3x^2 - 4 = 8$.
- **Turning points:** solve $\dfrac{dy}{dx} = 0$ for x, then put each x back into the ORIGINAL equation for y.
- **Nature:** $\dfrac{d^2y}{dx^2} < 0$ is a maximum, > 0 is a minimum.

Answer shapes (marked by **method marks** — M1 for a correct method, A1 for the correct answer): the derivative itself is a method mark, and setting it to zero is another — so write both lines down.

Method — Multiply by the power, then drop the power by one

Answer shapes (marked by method marks — M1 for a correct method, A1 for the correct answer): the derivative...



$$\frac{dy}{dx} = \text{gradient of the tangent}$$

A tangent drawn on a curve, showing the gradient at a point

2 Practice

Practice 2.1 ▪ 3 marks

Differentiate $y = 3x^4 - 5x^2 + 7$.

Solution 2.1

Method: Multiply by the power, then drop the power by one — p.4

Worked steps

- $3x^4 \to 12x^3$, $-5x^2 \to -10x$, and the 7 vanishes M1 M1

$$\frac{dy}{dx} = \mathbf{12x^3 - 10x}$$ A1

Practice 2.2 ▪ 3 marks

Find the gradient of the curve $y = x^3 - 4x$ at the point where $x = 2$.

Solution 2.2

Method: Multiply by the power, then drop the power by one — p.4

Worked steps

- $\dfrac{dy}{dx} = 3x^2 - 4$ **M1** **A1**
- substitute $x = 2$: $3(4) - 4$

$$= \mathbf{8}$$ **A1**

Practice 2.3 ▪ 3 marks

Find the coordinates of the turning point of $y = x^2 - 6x + 5$.

Solution 2.3

Method: Multiply by the power, then drop the power by one — p.4

Worked steps

- $\dfrac{dy}{dx} = 2x - 6$, and set it to zero **M1**
- $x = 3$ **A1**
- substitute into the original: $9 - 18 + 5$

$$= (\mathbf{3},\ -\mathbf{4})$$ **A1**

3 Exam-style

Exam-style 3.1 ▪ 8 marks

A curve has equation $y = 2x^3 - 9x^2 + 12x$.

Exam-style 3.1 (a) ▪ 2 marks

Find $\frac{dy}{dx}$.

Solution 3.1 (a)

Method: Multiply by the power, then drop the power by one — p.4

Worked steps

- differentiate term by term **M1**

$$\frac{dy}{dx} = \mathbf{6x^2 - 18x + 12}$$ **A1**

Exam-style 3.1 (b) ▪ 4 marks

Find the coordinates of the two turning points.

Solution 3.1 (b)

Worked steps

- set it to zero: $6x^2 - 18x + 12 = 0$, so $x^2 - 3x + 2 = 0$ **M1**
- $(x-1)(x-2) = 0$, so $x = 1$ or $x = 2$ **A1**
- $y(1) = 2 - 9 + 12 = 5$ and $y(2) = 16 - 36 + 24 = 4$ **M1**

$$\mathbf{(1,\ 5)} \text{ and } \mathbf{(2,\ 4)}$$ **A1**

Exam-style 3.1 (c) ▪ 2 marks

Determine, with a reason, which turning point is a maximum and which is a minimum.

Solution 3.1 (c)

Worked steps

- $\dfrac{d^2y}{dx^2} = 12x - 18$ M1
- at $x = 1$ this is -6, which is negative, so $(1,\ 5)$ is a **maximum**; at $x = 2$ it is $+6$, so $(2,\ 4)$ is a **minimum** A1