Exercise walk-through

Graphs of functions ▪ E2.10

eXercise

You must be able to

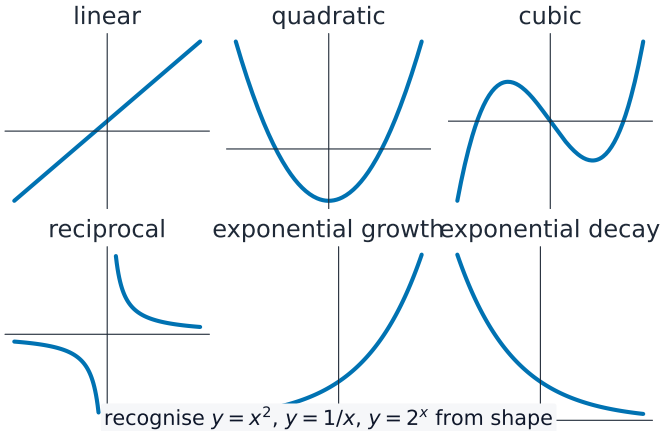
- complete a table of values for a function
- identify the roots and the y-intercept of a graph
- use a graph to solve an equation

Method — Tables, roots and intersections

- **Table:** for $y = x^2 - 3$, when $x = 2$, $y = 4 - 3 = 1$.
- **Roots 根**: where $y = 0$ (the graph crosses the x-axis).
- **Solve from a graph:** the **intersection 交点** of a line and a curve solves the two together.

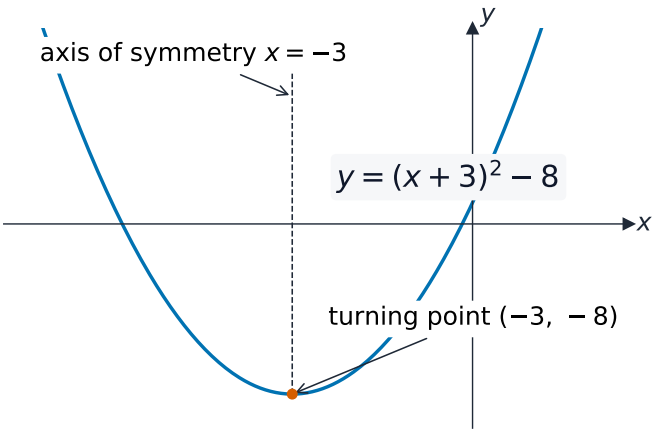
Answer shapes: **M1** a correct method, **A1** the correct answer.

Method — Tables, roots and intersections



The basic graph shapes

Method — Tables, roots and intersections



The quadratic is one key function graph

Method — Reading a graph, and solving with a line

The **roots** are where the curve crosses the x-axis, so they are the solutions of $y = 0$. The y-**intercept** is the value at $x = 0$. The **turning point** of $y = ax^2 + bx + c$ lies on the line of symmetry $x = -\dfrac{b}{2a}$; substitute that x back to get its y.

Solving with a drawn line. To solve an equation using a drawn curve, rearrange it so that one side is exactly the curve's equation; the other side is then the line to draw, and the solutions are the x-coordinates where the curve and line cross.

Worked example. The curve $y = x^2 - 2x - 3$ is drawn. Use it to solve $x^2 - 2x - 8 = 0$.

1. Rearrange so the left side matches the drawn curve: $x^2 - 2x - 8 = 0$ becomes $x^2 - 2x - 3 = 5$.
2. So draw the horizontal line $y = 5$.
3. Read the x-coordinates where the curve meets it: $x = -2$ and $x = 4$.
4. Check by factorising: $(x - 4)(x + 2) = 0$, which agrees.

Method — Reading a graph, and solving with a line

Sketching. Give the shape (a positive x^2 term opens upwards, a negative one opens downwards), the roots, the y-intercept and the turning point. For a cubic, the shape rises from bottom-left to top-right when the x^3 term is positive, and the roots are still where it crosses the x-axis.

Practice 2.1 ▪ 1 mark

For $y = x^2 - 4$, find the value of y when $x = 3$.

Solution 2.1

Worked steps

$y = 3^2 - 4 = 5$ B1.

Practice 2.2 ▪ 1 mark

Write down the y-intercept of $y = 2x^2 - 5$.

Solution 2.2

Worked steps

the y-intercept is at $x = 0$: $y = -5$ B1.

Practice 2.3 ▪ 2 marks

Find the roots of $y = x^2 - 9$.

Solution 2.3

Worked steps

$x^2 - 9 = 0 \rightarrow (x+3)(x-3) = 0 \rightarrow x = 3$ or $x = -3$ M1 A1 *for $x^2 = 9$ or factors; for both.*

Practice 2.4 ▪ 3 marks

Find the coordinates of the turning point of $y = x^2 - 6x + 5$.

Solution 2.4

Method: Reading a graph, and solving with a line

Worked steps

$x = -\dfrac{-6}{2} = 3$ M1; $y = 9 - 18 + 5 = -4$ M1; the turning point is $(3, -4)$ A1.

Practice 2.5 ▪ 1 mark

State the line you would draw on the graph of $y = x^2 + x$ to solve $x^2 + x - 4 = 0$.

Solution 2.5

Method: Reading a graph, and solving with a line

Worked steps

$y = 4$ **B1**.

Exam-style 3.1 ▪ 1 mark

$y = x^2 - 2x - 3$.

(a) Complete the value of y when $x = -2$.

(b) Find the roots of $x^2 - 2x - 3 = 0$.

Solution 3.1

Worked steps

(a) $y = (-2)^2 - 2(-2) - 3 = 4 + 4 - 3 = 5$ B1. (b) $(x-3)(x+1) = 0 \to x = 3$ or $x = -1$ M1 A1 *for the factors; for both.*

Exam-style 3.2 ▪ 3 marks

The graph of $y = x^2 - 2x - 3$ is drawn. By solving a quadratic, find the x-values where the graph meets the line $y = 5$.

Solution 3.2

Method: Reading a graph, and solving with a line

Worked steps

set $x^2 - 2x - 3 = 5 \rightarrow x^2 - 2x - 8 = 0 \rightarrow (x-4)(x+2) = 0 \rightarrow x = 4$ or $x = -2$

M1 M1 A1 *for* $x^2 - 2x - 8 = 0$*; for the factors; for both.*

Exam-style 3.3 ▪ 3 marks

The curve $y = x^2 - 4x + 1$ is to be drawn for $-1 \leqslant x \leqslant 5$.

(a) Complete the table of values.

x	-1	**0**	**1**	**2**	**3**	**4**	**5**
y							

(b) Write down the coordinates of the turning point of the curve.

(c) Use the curve to solve $x^2 - 4x + 1 = 0$, giving your answers correct to 1 decimal place.

Exam-style 3.3 ▪ 3 marks

(d) By drawing a suitable straight line, explain how you would solve $x^2 - 4x - 2 = 0$, and give the equation of that line.
(e) Solve $x^2 - 4x - 2 = 0$ algebraically, giving your answers correct to 2 decimal places.

Solution 3.3

Method: Reading a graph, and solving with a line

Worked steps

(a) $6, 1, -2, -3, -2, 1, 6$ **B3** *two marks for five correct, one for three correct.*

(b) $x = -\dfrac{-4}{2} = 2$ **M1**; $y = 4 - 8 + 1 = -3$ **M1**; the turning point is $(2, -3)$ **A1**.

(c) The curve crosses the x-axis at about $x = 0.3$ and $x = 3.7$ **B1** **B1**.

(d) Rearranging, $x^2 - 4x - 2 = 0$ becomes $x^2 - 4x + 1 = 3$ **B1**, so draw the line $y = 3$ and read where it meets the curve **B1**.

(e) $a = 1$, $b = -4$, $c = -2$, so $x = \dfrac{4 \pm \sqrt{16 + 8}}{2}$ **M1** **M1** *for* $\sqrt{24}$; $\sqrt{24} = 4.899$ **M1**; $x = 4.45$ or $x = -0.45$ **A1**.