

Exercise walk-through

C8.3

Probability of combined events

IGCSE Mathematics

You must be able to

- Use sample spaces and probability trees for independent stages
- Multiply along paths and add exclusive paths
- Distinguish exactly one from at least one outcome

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The method

Method — Key knowledge: Probability of combined events

For two independent stages, multiply along a tree path and add probabilities of different paths for the required event. With replacement, the second draw has the same probabilities as the first. Label probabilities beside branches and outcomes at their ends. A sample-space table lists every ordered pair once. In a two-set Venn diagram, add disjoint regions and count overlap once.

Exercises 2.1·2.3

2.1 Two fair coins are tossed. List their ordered sample space. [2]

2.2 For two fair coins, find the probability of exactly one head. [2]

2.3 A bag has 3 green and 1 white counter. Two draws are made with replacement. Find the probability of two whites. [2]

Exercises 2.1·2.3

2.1 Two fair coins are tossed. List their ordered sample space.

[2]

2.1

- Each sequence occurs once: **HH, HT, TH, TT**.

2.2 For two fair coins, find the probability of exactly one head.

[2]

2.2

- Two of four outcomes qualify: $p = 2/4 = 1/2$.

2.3 A bag has 3 green and 1 white counter. Two draws are made with replacement. Find the probability of two whites.

[2]

2.3

- White stays $1/4$: $p = (1/4)^2 = 1/16$.

Exercise 2.4

2.4 A fair coin and a fair die are used. Find the probability of a tail and a number above 4. [2]

3 Exam-style questions

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2.4

- $p = (1/2)(2/6) = \mathbf{1/6}$, using independence.

Exercise 3.1 (a)

3.1 Apply the probability of combined events methods below. Show working and include units where appropriate.

(a) A bag has 1 red and 2 blue counters. Draw twice with replacement. Draw and label a probability tree. [2]

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(a)

- **First branches R $\frac{1}{3}$, B $\frac{2}{3}$; each splits again into R $\frac{1}{3}$, B $\frac{2}{3}$. End labels are RR, RB, BR, BB.**

Exercise 3.1 (b)

3.1 Apply the probability of combined events methods below. Show working and include units where appropriate.

(b) Find the probability of exactly one red.

[2]

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3.1 Apply the probability of combined events methods below. Show working and include units where appropriate.

(b) Find the probability of exactly one red.

[2]

(b)

- $p = (1/3)(2/3) + (2/3)(1/3) = 4/9.$

Exercise 3.1 (c)

3.1 Apply the probability of combined events methods below. Show working and include units where appropriate.

(c) Find the probability of at least one red, and check using the complement. [2]

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(c)

- $p = 1 - (2/3)^2 = \mathbf{5/9}$; also $1/9 + 4/9 = 5/9$.

Exercise 3.2 (a)

3.2 A spinner has 3 equally sized red sectors and 1 blue sector. It is spun twice independently.

(a) Find the probability of exactly one blue, showing both paths. [3]

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(a)

- Each stage has $P(B) = 1/4$ and $P(R) = 3/4$.
- Add BR and RB: $p = (1/4)(3/4) + (3/4)(1/4)$, giving $p = \mathbf{3/8}$.

Exercise 3.2 (b)

3.2 A spinner has 3 equally sized red sectors and 1 blue sector. It is spun twice independently.

(b) Find the probability of at least one blue using a complement. Explain why it exceeds the answer to part (a). [3]

Exercise 3.2 (b)

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(b) Find the probability of at least one blue using a complement. Explain why it exceeds the answer to part (a). [3]

(b)

- Complement is two reds: $q = 1 - (3/4)^2$, so $q = \mathbf{7/16}$.
- It also includes BB, probability $1/16$, which exactly-one excludes.