

Exercise walk-through

C8.2

Relative and expected frequencies

IGCSE Mathematics

You must be able to

- Estimate probability from relative frequency
- Use an estimated probability to predict an expected count
- Explain sampling variation and limitations of predictions

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The method

Method — Key knowledge: Relative and expected frequencies

Relative frequency is observed successes divided by trials; it estimates probability. Expected frequency is probability multiplied by number of trials. Random selection gives each equally eligible member a fair chance. A fair device has the stated equally likely outcomes; bias changes those probabilities. Larger experiments usually give more stable estimates, but neither an estimate nor an expected count guarantees the next result.

Exercises 2.1·2.3

2.1 A coin shows heads 44 times in 80 tosses. Estimate the probability of heads. [2]

2.2 An event has probability 0.2. Find its expected frequency in 150 trials. [2]

2.3 A fair die is rolled 300 times. Find the expected number of sixes. [2]

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2.1

- $p \approx 44/80 = \mathbf{0.55}$.
-

2.2 An event has probability 0.2. Find its expected frequency in 150 trials. [2]

2.2

- $E = np = 150(0.2) = \mathbf{30}$.
-

2.3 A fair die is rolled 300 times. Find the expected number of sixes. [2]

2.3

- $E = np = 300(1/6) = \mathbf{50}$.

Exercise 2.4

2.4 Explain why 20 heads in 30 tosses does not prove a coin is biased.

[2]

3 Exam-style questions

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3 Exam-style questions

2.4

- **Random variation can give uneven counts in a small sample; more trials are needed to judge the long-run pattern.**

Exercise 3.1 (a)

3.1 Apply the relative and expected frequencies methods below. Show working and include units where appropriate.

- (a) A factory checks 240 items and finds 18 faulty. Estimate the fault probability. [2]

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(a)

- $p \approx 18/240 = \mathbf{0.075}$.

Exercise 3.1 (b)

3.1 Apply the relative and expected frequencies methods below. Show working and include units where appropriate.

(b) Use the estimate to find the expected faulty count in 800 items. [2]

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- $E = np = 800(0.075) = \mathbf{60}.$

Exercise 3.1 (c)

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(c) Explain one limitation of this prediction.

[2]

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(c)

- **The rate is an estimate and can change; the actual count may differ** even if production stays similar.

Exercise 3.2 (a)

3.2 Machine A produces 8 faulty items in a sample of 100. Machine B produces 21 faulty items in a sample of 300.

(a) Estimate each fault probability and state which machine has the lower estimated rate. [3]

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(a)

- $p_A = 8/100 = \mathbf{0.08}$ and $p_B = 21/300 = \mathbf{0.07}$.
- B has the lower estimated rate, although the samples do not prove its true rate is lower.

Exercise 3.2 (b)

3.2 Machine A produces 8 faulty items in a sample of 100. Machine B produces 21 faulty items in a sample of 300.

(b) Pool both samples to estimate the fault probability. Predict the expected number faulty in 1200 items from the same mixture and state one limitation. [3]

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(b)

- Pooled estimate $p = (8 + 21)/(100 + 300) = \mathbf{29/400}$.
- Expected count $E = np = 1200(29/400) = \mathbf{87}$.
- Accept sampling variation or a changed production mixture/rate as a limitation.