

Exercise walk-through

C5.4

Surface area and volume

IGCSE Mathematics

You must be able to

- Find prism, cylinder, cone, pyramid and sphere volumes
- Calculate surface area from the exposed faces
- Use perpendicular height for volume and slant height where area requires it

1

The method

Method — Key knowledge: Surface area and volume

Volume measures space inside; surface area totals exposed faces. Cuboid: $V = lwh$, $S = 2(lw + lh + wh)$. Prism: $V = Ah$, where A is cross-section area and h is length. Cylinder: $V = \pi r^2 h$, curved area $2\pi rh$; add circular ends when needed. Cone: $V = \pi r^2 h/3$, curved area πrl , using slant height l . Pyramid: $V = Ah/3$. Sphere: $V = 4\pi r^3/3$, $S = 4\pi r^2$.

Exercises 2.1·2.3

2.1 Find the volume and surface area of a $5 \times 3 \times 2$ cm cuboid. [2]

2.2 A triangular prism has cross-section base 6 cm and height 4 cm; length is 9 cm. Find its volume. [2]

2.3 A pyramid has base area 30 cm^2 and perpendicular height 8 cm. Find its volume. [2]

Exercises 2.1·2.3

2.1 Find the volume and surface area of a $5 \times 3 \times 2$ cm cuboid.

[2]

2.1

- $V = 5(3)(2) = \mathbf{30} \text{ cm}^3$; $S = 2(15 + 10 + 6) = \mathbf{62} \text{ cm}^2$.
-

2.2 A triangular prism has cross-section base 6 cm and height 4 cm; length is 9 cm. Find its volume.

[2]

2.2

- $A = 6(4)/2 = 12$; $V = Ah = 12(9) = \mathbf{108} \text{ cm}^3$.
-

2.3 A pyramid has base area 30 cm^2 and perpendicular height 8 cm. Find its volume.

[2]

2.3

- $V = Ah/3 = 30(8)/3 = \mathbf{80} \text{ cm}^3$.

Exercise 2.4

2.4 Find the volume and surface area of a sphere of radius 3 cm in terms of π . [2]

3 Exam-style questions

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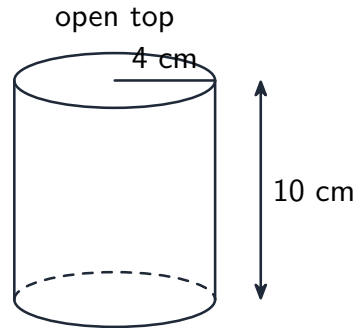
3 Exam-style questions

2.4

- $V = 4\pi(3)^3/3 = \mathbf{36\pi \text{ cm}^3}$; $S = 4\pi(3)^2 = \mathbf{36\pi \text{ cm}^2}$.

Setup for exercise 3.1

3.1 Apply the surface area and volume methods below. Show working and include units where appropriate.



Exercise 3.1 (a)

- (a) An open-top cylindrical container has radius 4 cm and height 10 cm. Find its volume in terms of π . [2]

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(a)

- $V = \pi r^2 h = \pi(4)^2(10) = \mathbf{160\pi \text{ cm}^3}$.

Exercise 3.1 (b)

- (b) Find its exposed area, including its circular base but not its open top. [2]

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(b)

- $S = 2\pi rh + \pi r^2 = 80\pi + 16\pi = \mathbf{96\pi} \text{ cm}^2.$

Exercise 3.1 (c)

(c) A cone has the same radius and height. Find its volume, and compare it with the cylinder. [2]

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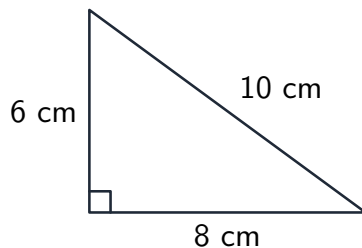
(c) A cone has the same radius and height. Find its volume, and compare it with the cylinder. [2]

(c)

- $V_{\text{cone}} = \pi r^2 h / 3 = 160\pi / 3 \text{ cm}^3$, **one third** of the cylinder volume.

Setup for exercise 3.2

3.2 A triangular prism is 10 cm long. Its right-triangle cross-section has perpendicular sides 6 cm and 8 cm and hypotenuse 10 cm.



cross-section; prism length 10 cm

not to scale

Exercise 3.2 (a)

(a) Find the prism volume.

[3]

Exercise 3.2 (a)

(a) Find the prism volume.

[3]

(a)

- Use $A = bh/2$ for the triangular cross-section.
- $A = 6(8)/2 = 24 \text{ cm}^2$.
- Then $V = Al = 24(10) = \mathbf{240} \text{ cm}^3$.

Exercise 3.2 (b)

(b) Find its total surface area, explaining how the two ends and three rectangular faces are included. [3]

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(b)

- Two ends give $S_e = 2(24) = 48 \text{ cm}^2$.
- Rectangular faces give $S_r = 10(6 + 8 + 10) = 240 \text{ cm}^2$.
- Total $S = S_e + S_r = \mathbf{288 \text{ cm}^2}$.