

Exercise walk-through

C4.7

Circle theorems

IGCSE Mathematics

You must be able to

- Identify a diameter and an angle in a semicircle
- Use the perpendicular tangent-radius property
- Combine a circle fact with triangle-angle sums and give reasons

1

The method

Method — Key knowledge: Circle theorems

Core circle theorems use two facts: an angle at the circumference standing on a diameter is 90° , and a tangent is perpendicular to the radius at the point of contact. Confirm that the line is actually a diameter or tangent before applying a theorem. Then combine with triangle or straight-line angle sums, stating each reason.

Exercises 2.1·2.3

2.1 AB is a diameter and C is a different point on the circle. Find angle ACB and give a reason. [2]

2.2 In that triangle, angle ABC is 61° . Find angle CAB. [2]

2.3 OT is a radius and PT a tangent at T. Find angle OTP with a reason. [2]

Exercises 2.1·2.3

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2.1

- 90° , angle in a semicircle.

2.2 In that triangle, angle ABC is 61° . Find angle CAB. [2]

2.2

- $A = 180 - 90 - 61 = \mathbf{29^\circ}$, using the triangle sum.

2.3 OT is a radius and PT a tangent at T. Find angle OTP with a reason. [2]

2.3

- 90° , tangent perpendicular to radius at contact.

Exercise 2.4

2.4 In triangle OTP, angle OPT is 27° . Find angle TOP.

[2]

3 Exam-style questions

Exercise 2.4

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[2]

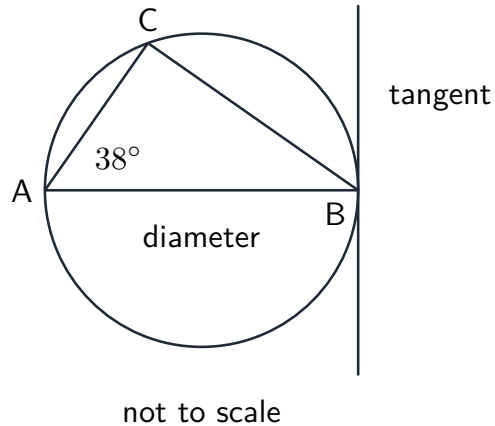
3 Exam-style questions

2.4

- $O = 180 - 90 - 27 = \mathbf{63^\circ}$, using the triangle sum.

Setup for exercise 3.1

3.1 Apply the circle theorems methods below. Show working and include units where appropriate.



Exercise 3.1 (a)

- (a) A circle has diameter AB ; C lies on its circumference. Angle CAB is 38° . Find angle ACB with a reason. [2]

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(a)

- 90° , angle in a semicircle.

Exercise 3.1 (b)

(b) Find angle ABC with a reason.

[2]

Exercise 3.1 (b)

(b) Find angle ABC with a reason.

[2]

(b)

- $B = 180 - 90 - 38 = \mathbf{52^\circ}$, triangle angle sum.

Exercise 3.1 (c)

(c) The tangent at B meets AB extended. Find its angle with BA, giving a reason. [2]

Exercise 3.1 (c)

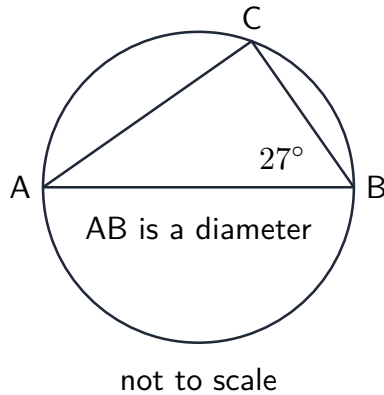
(c) The tangent at B meets AB extended. Find its angle with BA, giving a reason. [2]

(c)

- 90° ; BA lies along the radius at B, and tangent is perpendicular to radius.

Setup for exercise 3.2

3.2 AB is a diameter of a circle. C is on the circumference and angle $ABC = 27^\circ$.



Exercise 3.2 (a)

- (a) Find angles ACB and CAB , giving the circle fact and triangle fact used. [3]

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(a)

- $\angle ACB = \mathbf{90^\circ}$ because an angle in a semicircle is a right angle.
- Triangle angles sum to 180° , so $\angle CAB = 180 - 90 - 27 = \mathbf{63^\circ}$.

Exercise 3.2 (b)

(b) In a separate circle with centre O, PT is tangent at T. Angle $OPT = 32^\circ$. Find angle OTP and angle POT, giving reasons. [3]

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(b)

- $\angle OTP = 90^\circ$ because a tangent is perpendicular to the radius at contact.
- Triangle sum gives $\angle POT = 180 - 90 - 32$, so $\angle POT = 58^\circ$.