

Exercise walk-through

C2.7

Sequences

IGCSE Mathematics

You must be able to

- Continue sequences and identify their term-to-term rule
- Find linear, simple quadratic and simple cubic n th terms
- Check whether a proposed term has an integer index

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The method

Method — Key knowledge: Sequences

A term-to-term rule tells how to move to the next term; an n th-term rule finds any term directly. For a linear sequence with first term a and difference d , $u_n = a + (n - 1)d$. For simple quadratic or cubic sequences, compare with n^2 or n^3 and look for a constant addition or multiplication. Check a proposed rule against several terms.

Exercises 2.1·2.3

2.1 Write the next two terms of $5, 9, 13, 17, \dots$ [2]

2.2 Find the n th term of $6, 11, 16, 21, \dots$ [2]

2.3 Find the n th term of $3, 6, 11, 18, \dots$ [2]

Exercises 2.1·2.3

2.1 Write the next two terms of 5, 9, 13, 17, [2]

2.1

- Add 4 each time: **21, 25.**
-

2.2 Find the n th term of 6, 11, 16, 21, [2]

2.2

- $u_n = 6 + 5(n - 1) = \mathbf{5n + 1.}$
-

2.3 Find the n th term of 3, 6, 11, 18, [2]

2.3

- Subtract squares 1, 4, 9, 16: the difference is 2, so $u_n = n^2 + 2.$

Exercise 2.4

2.4 Find the fifth term of $u_n = n^3 - 1$.

[2]

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[2]

2.4

- $u_5 = 5^3 - 1 = 125 - 1 = \mathbf{124}$.

Exercise 3.1 (a)

3.1 Apply the sequences methods below. Show working and include units where appropriate.

(a) A pattern has 7, 11, 15, 19, ... tiles. Find its n th term.

[2]

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3.1 Apply the sequences methods below. Show working and include units where appropriate.

(a) A pattern has 7, 11, 15, 19, ... tiles. Find its n th term. [2]

(a)

- $u_n = 7 + 4(n - 1) = 4n + 3.$

Exercise 3.1 (b)

3.1 Apply the sequences methods below. Show working and include units where appropriate.

(b) Find the number of tiles in pattern 30.

[2]

Exercise 3.1 (b)

3.1 Apply the sequences methods below. Show working and include units where appropriate.

(b) Find the number of tiles in pattern 30. [2]

(b)

- $u_{30} = 4(30) + 3 = \mathbf{123}.$

Exercise 3.1 (c)

3.1 Apply the sequences methods below. Show working and include units where appropriate.

(c) Can a pattern in this sequence have 100 tiles? Explain.

[2]

Exercise 3.1 (c)

3.1 Apply the sequences methods below. Show working and include units where appropriate.

(c) Can a pattern in this sequence have 100 tiles? Explain. [2]

(c)

- $4n + 3 = 100$ gives $n = 97/4 = 24.25$. **No**, a pattern number must be a positive integer.

Exercise 3.2 (a)

3.2 A sequence begins $2, 5, 10, 17, \dots$

(a) Find the next two terms and an expression for its n th term.

[3]

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3.2 A sequence begins 2, 5, 10, 17, . . .

(a) Find the next two terms and an expression for its n th term.

[3]

(a)

- These are one more than successive squares.
- The next terms are **26 and 37**, and $u_n = \mathbf{n^2 + 1}$.

Exercise 3.2 (b)

3.2 A sequence begins $2, 5, 10, 17, \dots$

(b) Decide whether 50 and 60 are terms of this sequence. Show how the term number determines each decision. [3]

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(b)

- For 50, $n^2 = 49$ gives $n = 7$, so **50 is a term**.
- For 60, $n^2 = 59$ lies strictly between 7^2 and 8^2 ; there is no integer index, so **60 is not a term**.