

Probability

IGCSE Mathematics ■ Topic 8

IGCSE Mathematics



What we will cover

- 1 The probability scale
- 2 Relative & expected frequency
- 3 Combined events
- 4 Sample space diagrams
- 5 Venn diagrams
- 6 Tree diagrams



the probability scale

Parts marked (Extended) are tested only on the Extended paper; everything else is for both levels.

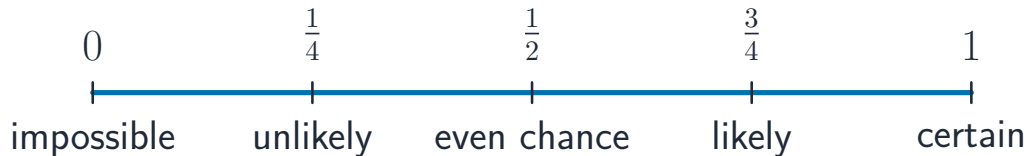


The probability scale

Probability runs from 0 (impossible) to 1 (certain), with $\frac{1}{2}$ an even chance.

For equally likely **outcomes**, $P(\text{event}) = \frac{\text{favourable outcomes}}{\text{total outcomes}}$, and the probability it does **not** happen is $P(A') = 1 - P(A)$.

The probability scale, in detail



Probability runs from 0 (impossible) to 1 (certain), with $\frac{1}{2}$ an even chance.

Relative and expected frequency

relative frequency

$$\frac{\text{times it happened}}{\text{total trials}}$$

better with more trials

expected frequency

$P \times n$; e.g. $\frac{1}{6} \times 300 = 50$ sixes

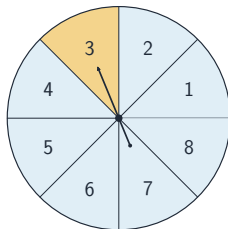
Combined events

AND – multiply

both happen, when
independent

OR – add

either happens, when
mutually exclusive



Eight identical sectors, so every number is **equally likely**.

$$P(3) = \frac{\text{favourable outcomes}}{\text{total outcomes}} = \frac{1}{8}$$

A fair spinner: eight identical sectors, so each number is an equally likely outcome.

The two rules, and the three diagrams

the AND rule

for **independent** events, $P(A \text{ and } B) = P(A) \times P(B)$

the OR rule

for **mutually exclusive** events, $P(A \text{ or } B) = P(A) + P(B)$

a sample space diagram

a grid of all equally likely outcomes – best for two dice or two spinners

a Venn diagram

best when events **overlap**, and for reading conditional probabilities

a tree diagram

best for events **in sequence**: multiply along a branch, add between branches

random

every outcome has an equal chance; a **biased** spinner or die does not

Multiply **along** a tree branch, add **between** branches. Those two moves solve almost every tree question.

The three pictures, and when each helps

a sample space diagram

a **grid** of every equally likely outcome – best for two dice or two spinners

a Venn diagram

best when events **overlap**, and for reading conditional probabilities off

a tree diagram

best for events **in sequence** – two draws, or two stages

on a tree

multiply along a branch, **add between** branches

with replacement

the second stage's probabilities are the same as the first's

without replacement

they change – the denominator falls by one, and so does the count of whatever was drawn

Three pictures organise the work; choose one from the **shape of the question**: a grid for two simultaneous events, a tree for two in sequence, a Venn for overlap.



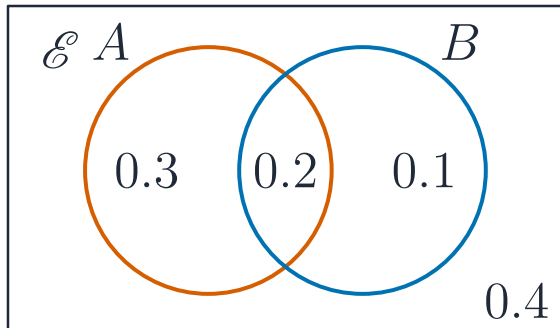
Sample space diagrams

		die 2					
+		1	2	3	4	5	6
die 1	1	2	3	4	5	6	7
	2	3	4	5	6	7	8
	3	4	5	6	7	8	9
	4	5	6	7	8	9	10
	5	6	7	8	9	10	11
	6	7	8	9	10	11	12

A **sample space diagram** lists every outcome in a grid. Two dice give 36 equally likely cells; count the ones you want.

the six highlighted cells all give a total of 7

Venn diagrams



A **Venn diagram** sorts outcomes into sets.

- $P(A \cap B)$ – in **both**
- $P(A \cup B)$ – in **either**

Worked example – conditional probability (Extended)

In a class of 30, 18 study French and, of those, 7 also study German. A French student is picked. Find the probability they study German.

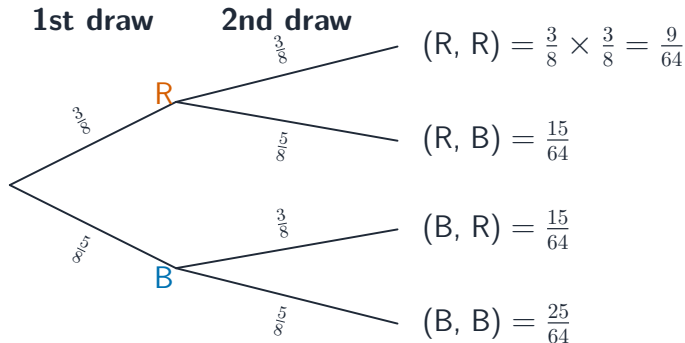
- **conditional probability** : the probability of one event **given that** another has happened
- look **only** at the 18 French students
- $P(\text{German} \mid \text{French}) = \frac{7}{18}$

The condition **replaces the denominator**: you are no longer choosing from 30 but from 18. Circle the region you are inside before writing any fraction.

Tree diagrams

On a **tree diagram** , **multiply** along branches & **add** the paths you want. Two reds (with replacement): $\frac{3}{8} \times \frac{3}{8} = \frac{9}{64}$.

Tree diagrams, in detail



On a **tree diagram** , **multiply** along branches & **add** the paths you want.

The probability scale, continued



Dice: the probability scale runs from impossible (0) to certain (1).

Exam tips

- Every probability lies between 0 and 1, and the probabilities of all the outcomes add to 1.
- For “**and**” (both events) *multiply* the probabilities; for “**or**” (either event) *add* them. On a tree diagram, multiply along the branches.
- Watch for **without replacement**: the second probability changes because one item has already been removed.
- Expected frequency = probability $\times$ number of trials.



Worked example – probability of 'not'

**A bag holds 3 red and 5 blue counters.
Find the probability of *not* drawing red.**

- Total counters = $3 + 5 = 8$.
- $P(\text{red}) = \frac{3}{8}$
- $P(\text{not red}) = 1 - P(\text{red}) = 1 - \frac{3}{8} = \frac{5}{8}$
- Check: there are 5 blue out of 8 – the same answer, as it must be.

$P(\text{not } A) = 1 - P(A)$. All probabilities lie between 0 and 1: a negative answer or one above 1 means you have made a slip.

Topic 8 – key takeaways

1

Scale

0 to 1; $P(A') = 1 - P(A)$

2

Expected

$P \times n$ over n trials

3

Combined

AND multiply, OR add

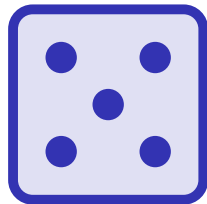
4

Trees

multiply along branches, add the paths

Check yourself

- 1 What does probability 0 mean?
- 2 If $P(A) = 0.3$, what is $P(A')$?
- 3 Do you add or multiply for AND?
- 4 What do you do along a tree branch?



Answers 1. impossible 2. 0.7 3. multiply 4. multiply the probabilities