

Transformations & Vectors

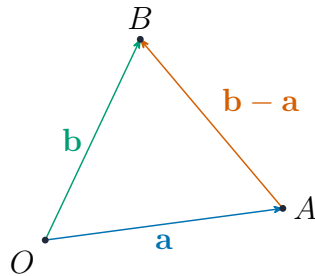
IGCSE Mathematics ■ Topic 7

IGCSE Mathematics



What we will cover

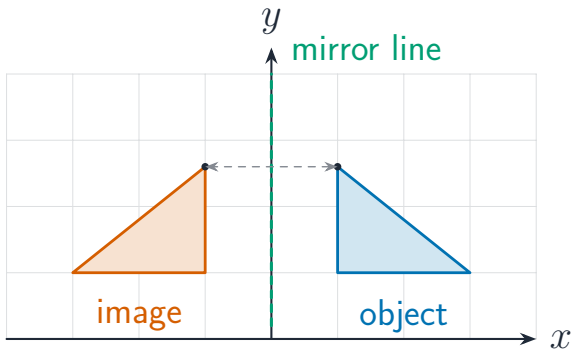
- 1 Reflection
- 2 Rotation
- 3 Enlargement
- 4 Translation
- 5 Vectors
- 6 Magnitude



vector geometry

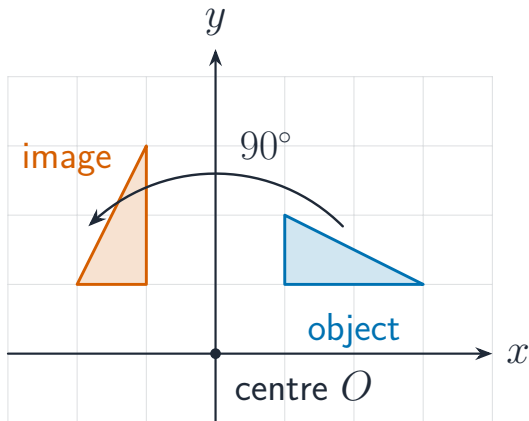


Reflection



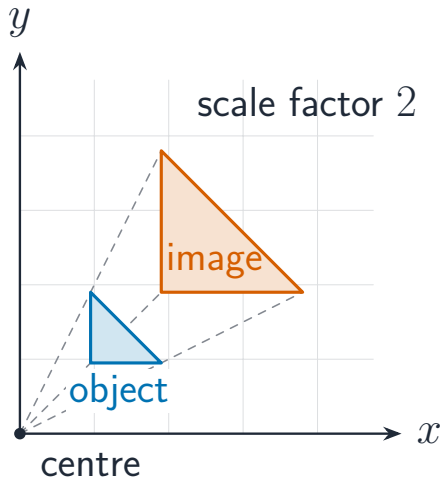
- To describe it, give the **equation of the mirror line** (for Core, a horizontal or vertical line; Extended allows any line such as $y = x$).

Rotation



- To describe it, give the centre, the angle, and the direction (clockwise or anticlockwise).

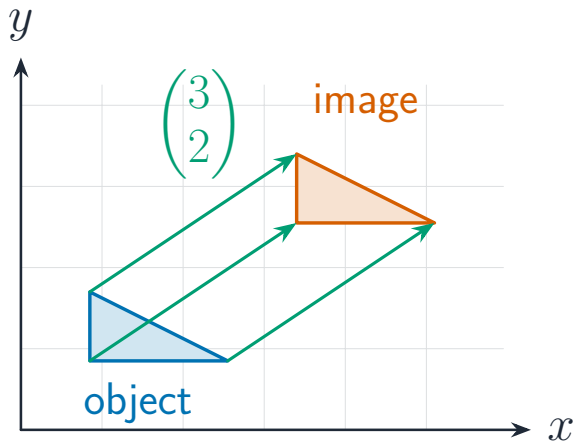
Enlargement



An **enlargement** scales every distance from a centre by k .

Give the **centre** & the **scale factor** k (k may be fractional or negative).

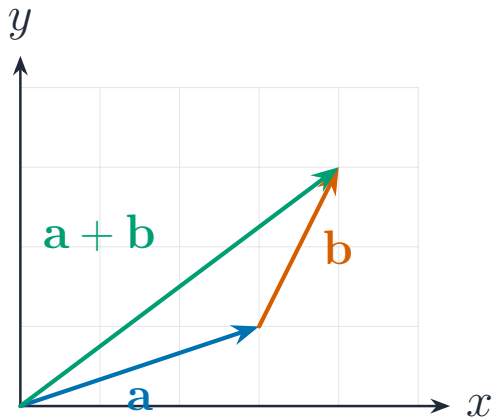
Translation



A **translation** slides the shape with no turning, by a **column vector** $\begin{pmatrix} x \\ y \end{pmatrix}$ (x across, y up).



Vectors



A **vector** has size & direction.

- add/subtract top & bottom separately

- $\times$ a **scalar** : multiply both

$$\begin{pmatrix} 3 \\ 1 \end{pmatrix} + \begin{pmatrix} 2 \\ -4 \end{pmatrix} = \begin{pmatrix} 5 \\ -3 \end{pmatrix}.$$

Two things you can do to a vector

Multiply by a scalar

To **multiply by a scalar** stretches the vector and keeps its direction – or reverses it if the scalar is negative.

Add them

Tip-to-tail, or add the components. $\vec{AB} + \vec{BC} = \vec{AC}$, whatever the route.

Parallel

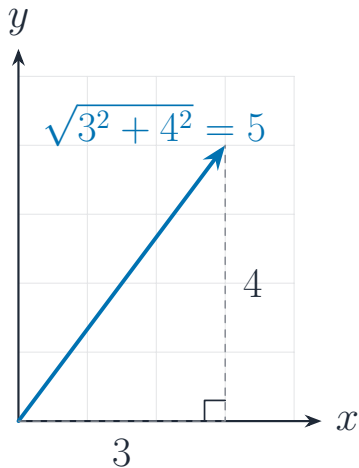
Two vectors are parallel exactly when one is a scalar multiple of the other – which is how a proof of collinearity is written.

Coplanar

Vectors are **coplanar** when they all lie in one plane, so each can be written from the other two.

To prove three points collinear, show two of the vectors between them are parallel *and* share a point.

Magnitude



The **magnitude** (length) uses Pythagoras:

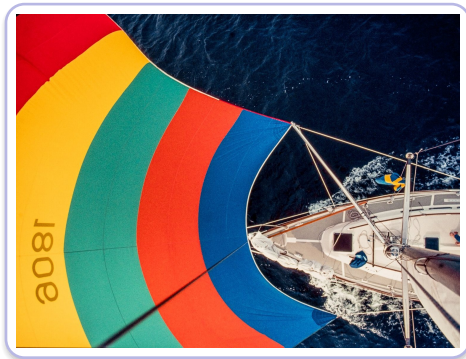
$$\left| \begin{pmatrix} x \\ y \end{pmatrix} \right| = \sqrt{x^2 + y^2}$$

Transformations



Geometric tiles are built by reflecting, rotating and translating shapes.

Vectors in two dimensions (Extended)



Forces like wind are vectors, with both size and direction.

Transformations, and the vector words

a transformation

changes the position or size of a shape: the original is the **object**, the result the **image**

what to state

reflection needs a **mirror line**; rotation needs a **centre, angle and direction**; enlargement a **centre and scale factor**; translation a **column vector**

a directed line segment

a vector drawn as an arrow – length is magnitude, direction is direction

a position vector

a vector from the **origin** to a point

multiplying by a scalar

changes the length and, if negative, reverses the direction – but never the line it lies along

collinear points

if $\vec{AB}$ is a scalar multiple of $\vec{BC}$ and they share B , then A , B and C lie on one line

Describing a transformation **fully** is what is marked. A reflection without its mirror line, or a rotation without its centre, is a half answer.

Exam tips

- Fully **describe** each transformation: a translation (a vector), a reflection (the mirror line), a rotation (centre, angle *and* direction), an enlargement (centre and scale factor).
- A **negative** scale factor turns the image upside down through the centre; a fractional one (between 0 and 1) makes it smaller.
- Add vectors tip-to-tail – add the top numbers, then the bottom numbers. The **magnitude** of a vector comes from Pythagoras on its components.
- $\overrightarrow{AB} = \mathbf{b} - \mathbf{a}$ (end minus start). Two vectors are **parallel** if one is a scalar multiple of the other.



Worked example – reflections

Reflect the point $(3, 2)$ in the y -axis, then in the x -axis.

- In the y -axis (the vertical one): only the sign of x changes.
- $(3, 2) \rightarrow (-3, 2)$
- In the x -axis (the horizontal one): only the sign of y changes.
- $(3, 2) \rightarrow (3, -2)$

Reflecting in the y -axis flips x ; in the x -axis flips y – the **opposite** letter to the axis named. Points **on** the mirror line never move.

Topic 7 – key takeaways

1

Reflection

flip over a mirror line

2

Rotation

centre, angle & direction

3

Enlargement

centre & scale factor k

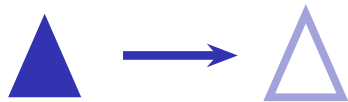
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Vectors

add componentwise; $|v| = \sqrt{x^2 + y^2}$

Check yourself

- 1 What must you give to describe a reflection?
- 2 What three things describe a rotation?
- 3 What kind of transformation is a slide?
- 4 Find $\left| \begin{pmatrix} 3 \\ 4 \end{pmatrix} \right|$.



Answers 1. the equation of the mirror line 2. centre, angle, direction 3. a translation
4. $\sqrt{9 + 16} = 5$