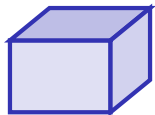


Mensuration

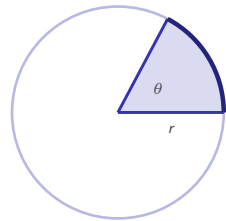
IGCSE Mathematics ■ Topic 5

IGCSE Mathematics



What we will cover

- 1 Units of measure
- 2 Area of basic shapes
- 3 Circles
- 4 Arcs & sectors
- 5 Solids: volume & surface
- 6 Compound shapes



arcs and sectors



Units of measure

- Length: $1 \text{ m} = 100 \text{ cm}$.
- Area: $1 \text{ m}^2 = 100^2 = 10\,000 \text{ cm}^2$.
- Volume: $1 \text{ m}^3 = 100^3 = 1\,000\,000 \text{ cm}^3$.
- Capacity: $1 \text{ litre} = 1000 \text{ cm}^3$, so $1 \text{ m}^3 = 1000 \text{ litres}$.



(to a smaller unit, multiply)

Converting units, squares and cubes included

length

$$1 \text{ m} = 100 \text{ cm} = 1000 \text{ mm}$$

area

$$\text{square the length factor: } 1 \text{ m}^2 = 100^2 = 10\,000 \text{ cm}^2$$

volume

$$\text{cube it: } 1 \text{ m}^3 = 100^3 = 1\,000\,000 \text{ cm}^3$$

capacity

$$1 \text{ litre} = 1000 \text{ cm}^3, \text{ and } 1 \text{ ml} = 1 \text{ cm}^3$$

mass

$$1 \text{ kg} = 1000 \text{ g}, 1 \text{ tonne} = 1000 \text{ kg}$$

the check

converting to a **smaller** unit always gives a **bigger** number

The commonest error in this whole topic is using the length factor for an area or a volume. Square it or cube it **every time**.

Area of basic shapes

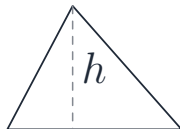
The **perimeter** is the distance all the way round a shape.

The **area** is the amount of flat space inside it.

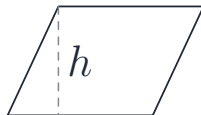
Area of basic shapes, in detail



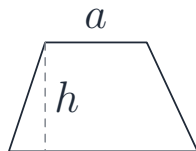
$$\ell \times w$$



$$\frac{1}{2} b h$$



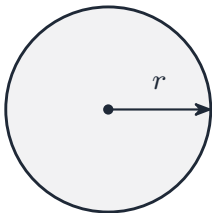
$$b h$$



$$\frac{1}{2} (a + b) h$$

The **perimeter** is the distance all the way round a shape.

Circles



$$\text{circumference} = \pi d$$

$$\text{area} = \pi r^2$$

$$(\text{diameter } d = 2r)$$

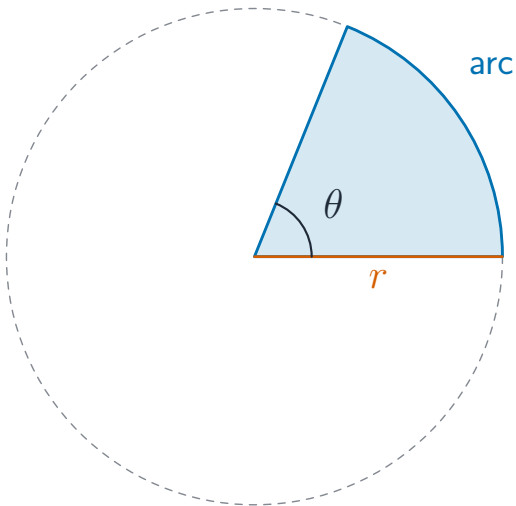
For a **circle** with **radius** r (and **diameter** $d = 2r$):

$$\text{circumference} = 2\pi r = \pi d, \quad \text{area} = \pi r^2.$$

The **circumference** is the distance round...

$$\text{circumference} = 2\pi \times 7 = 14\pi \text{ cm}, \quad \text{area} = \pi \times 7^2 = 49\pi \text{ cm}^2.$$

Arcs and sectors



A **sector** is the fraction $\frac{\theta}{360}$ of the circle.

$$\text{arc} = \frac{\theta}{360} \times 2\pi r$$

$$\text{sector area} = \frac{\theta}{360} \times \pi r^2$$

Areas, arcs and sectors

parallelogram

base $\times$ perpendicular height

trapezium

$\frac{1}{2}(a + b)h$, with a and b the two **parallel** sides

triangle

$\frac{1}{2}bh$, or $\frac{1}{2}ab \sin C$ when you have two sides and the angle between

arc length

$\frac{\theta}{360} \times 2\pi r$

sector area

$\frac{\theta}{360} \times \pi r^2$

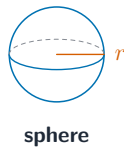
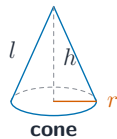
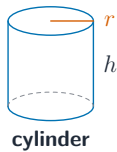
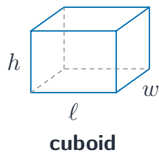
the two sectors

a small slice is a **minor sector** ; the large rest is a **major sector**

Both arc and sector formulas are the **whole circle scaled by $\theta/360$** . Learn one idea rather than two formulas.

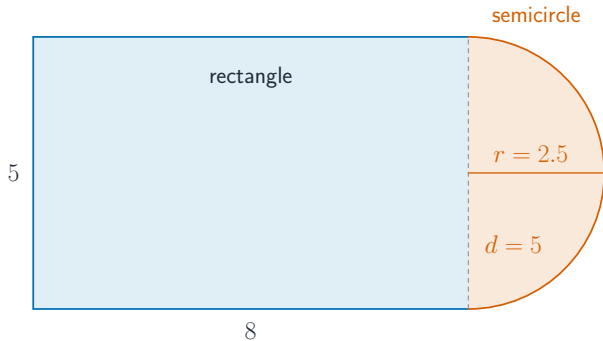


Solids: volume and surface area



- **prism** : cross-section $\times$ length
- **cylinder** : $\pi r^2 h$
- **cone** : $\frac{1}{3}\pi r^2 h$
- **sphere** : $\frac{4}{3}\pi r^3$

Compound shapes



Split a **compound shape** into parts you know, find each area (or volume), then **add** them up.

Surface area and volume of solids



The pyramids of Giza are square-based pyramids – a 3-D solid.

The curved surfaces

Cylinder

Curved surface area = $2\pi rh$ – unroll it and it is a rectangle of height h and width the circumference.

Cone

Curved surface = πrl , where l is the **slant height** – not the vertical height. Pythagoras connects the two.

Sphere and hemisphere

Surface $4\pi r^2$, volume $\frac{4}{3}\pi r^3$. A **hemisphere** is half the volume *plus* a flat circular face.

Frustum

A **frustum** is a cone with its top cut off: work out the whole cone and subtract the small one.

Check which height a formula wants. Slant height for the cone's surface, vertical height for its volume.

Compound shapes and parts of shapes



Stacked containers are cuboids; volume is length times width times height.

Exam tips

- Match the formula to the shape: the **area** of a circle is πr^2 , the **circumference** is πd (or $2\pi r$) – do not mix them up.
- Keep units consistent, and remember area units are squared and volume units cubed (e.g. $1 \text{ m}^2 = 10\,000 \text{ cm}^2$).
- For an **arc** or **sector**, take the fraction $\frac{\theta}{360}$ of the whole circumference or area.
- Surface area is the total of *all* the faces – draw the net if you are unsure. Leave an answer in terms of π only if the question allows it.



Worked example – converting areas

Convert 3 m^2 to cm^2 .

- $1 \text{ m} = 100 \text{ cm}$, so **square the conversion factor**:
- $1 \text{ m}^2 = 100^2 = 10\,000 \text{ cm}^2$
- $3 \text{ m}^2 = 3 \times 10\,000 = 30\,000 \text{ cm}^2$
- For volume, **cube** it:
 $1 \text{ m}^3 = 100^3 = 1\,000\,000 \text{ cm}^3$.

The classic error is $\times 100$.
Area needs the factor **squared**, **volume** needs it **cubed** – because area is length^2 and volume length^3 .

Topic 5 – key takeaways

1

Units

area $\times 100^2$, volume $\times 100^3$ per
metre

2

Circle

$$C = 2\pi r, \text{ area} = \pi r^2$$

3

Sector

the fraction $\theta/360$ of the circle

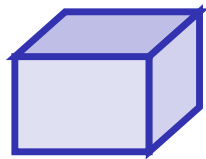
4

Solids

cylinder $\pi r^2 h$, sphere $\frac{4}{3}\pi r^3$

Check yourself

- 1 How many cm^2 are in 1 m^2 ?
- 2 What is the area of a circle of radius r ?
- 3 What fraction of a circle is a 90° sector?
- 4 What is the volume of a sphere of radius r ?



Answers 1. 10 000 2. πr^2 3. one quarter 4. $\frac{4}{3}\pi r^3$