

Geometry

IGCSE Mathematics ■ Topic 4

IGCSE Mathematics



What we will cover

- 1 Angles & angle facts
- 2 Parallel-line angles
- 3 Triangles & polygons
- 4 Circle parts
- 5 Circle theorems
- 6 Bearings & similar shapes



Angle types



acute
 $< 90^\circ$



right
 $= 90^\circ$



obtuse
 $90^\circ - 180^\circ$



reflex
 $180^\circ - 360^\circ$

- Angles at a point add up to 360° .
- Angles on a straight line add up to 180° .
- **Vertically opposite angles** (made by two crossing lines) are equal.

Naming an angle, and naming a triangle

By size

An **acute angle** is under 90° , a right angle exactly 90° , an **obtuse angle** between 90° and 180° , and a **reflex angle** more than 180° .

Equilateral

An **equilateral** triangle: three equal sides, three angles of 60° .

Isosceles

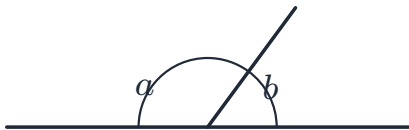
An **isosceles** triangle: two equal sides, and the angles opposite them are equal – the fact most angle-chasing questions turn on.

Scalene

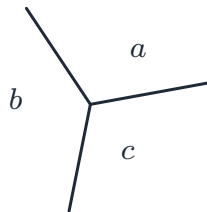
A **scalene** triangle: all three sides different, so no equal angles to use.

Name the triangle before chasing the angle. Which sides are equal is usually the missing step.

Angle facts



$$a + b = 180^\circ \text{ (straight line)}$$



$$a + b + c = 360^\circ \text{ (round a point)}$$

Angles on a line add to 180° ; around a point to 360° ; **vertically opposite** angles are equal.

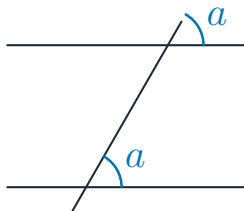
Always **give the reason**.

Parallel-line angles

- **Corresponding angles** (in matching positions, an “F” shape) are equal.
- **Alternate angles** (opposite sides of the crossing line, a “Z” shape) are equal.
- **Co-interior angles** (a “C” shape) add up to 180° ; we say they are **supplementary** .

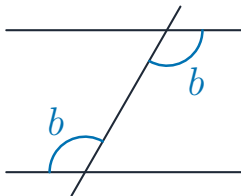
Parallel-line angles, in detail

Corresponding (F)



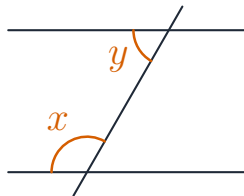
equal

Alternate (Z)



equal

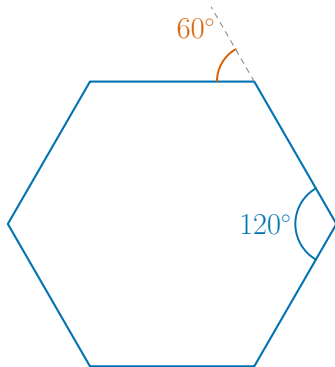
Co-interior (C)



$$x + y = 180^\circ$$

Corresponding (F) and alternate (Z) angles are equal; co-interior (C) angles add to 180° .

Triangles and polygons

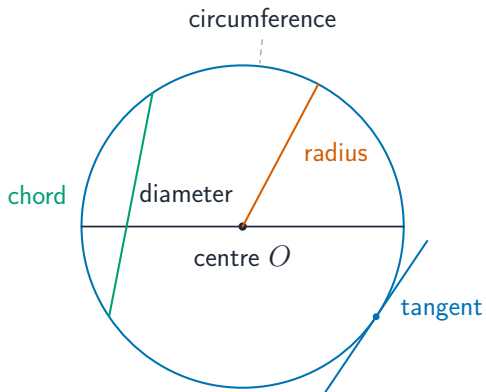


$$\text{interior} + \text{exterior} = 180^\circ$$

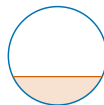
- A shape has **line symmetry** if a mirror line splits it into two matching halves.
- A shape has **rotational symmetry** if it fits onto itself as you turn it. The order is how many times it fits in one full turn.



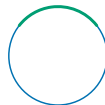
Circle parts



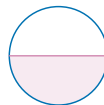
sector



segment



arc



semicircle

radius , **diameter** , **chord** , **tangent** ; a **sector** is a slice, a **segment** is cut off by a chord.

The parts of a circle, and the theorem names

radius

centre to edge (plural *radii*); the **diameter** is twice it

a chord

a straight line joining two points on the circle

an arc

part of the circumference; a **sector** is bounded by two radii, a **segment** by a chord

a tangent

touches at one point, and is **perpendicular** to the radius there

(Extended) equal chords

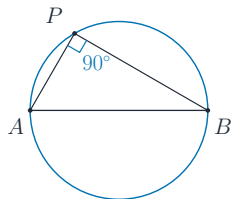
are the **same distance** from the centre

and

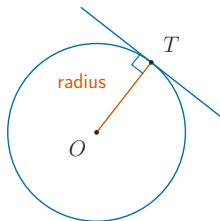
the **perpendicular bisector** of a chord passes through the centre; the **alternate segment theorem** links a tangent-chord angle to the angle in the other segment

Name the theorem you are using. “Angle in a semicircle is 90° ” scores where “it looks like a right angle” does not.

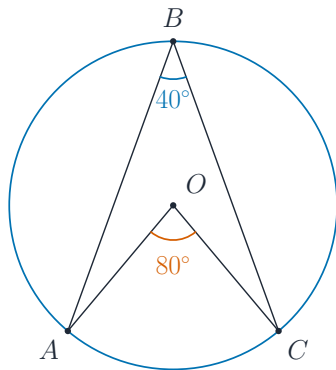
Circle theorems



angle in a
semicircle = 90°



tangent meets
radius = 90°

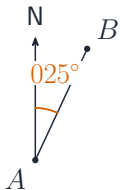


angle at centre = $2 \times$ angle at circumference

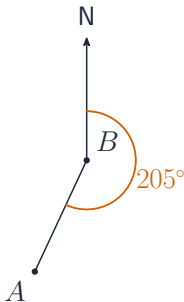
Angle in a **semicircle** = 90° ; tangent $\perp$ radius. The angle at the **centre** is **twice** the angle at the circumference.



Bearings



Bearing of B from A



Bearing of A from B

A **bearing** is measured clockwise from north, written with three digits. So due east is 090° and south-west is 225° .

Similar shapes

length
 $\times k$

area
 $\times k^2$

volume
 $\times k^3$

Similar shapes have the same angles & sides $\times$ a **scale factor** k . (**Congruent** = identical.)

Quadrilaterals, symmetry and solids

quadrilaterals

angles sum to 360° :
square, rectangle, rhombus, kite

symmetry

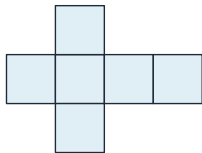
lines of reflection;
order of rotational symmetry

nets & solids

a net folds into a solid; add the faces for surface area

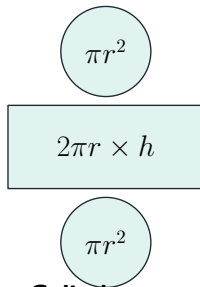
A **net** is the key to surface area – unfold the solid, find each face's area, add them.

Constructions, nets and solids



Cube net

6 equal square faces



Cylinder net

2 circles + a rectangle

Unfolding a solid gives its net. Add the areas of the pieces to get the surface area: a cylinder is $2\pi r^2 + 2\pi rh$.

Constructions, and the names of solids

to construct a triangle

draw the base with a ruler, then use **compasses** to swing the other two side lengths from each end

a scale drawing

shows a real object smaller or larger by a fixed scale, such as 1 cm to 5 m

prism and cylinder

the same cross-section all the way along; a cylinder is a circular prism

pyramid and cone

come to a **point**

sphere and hemisphere

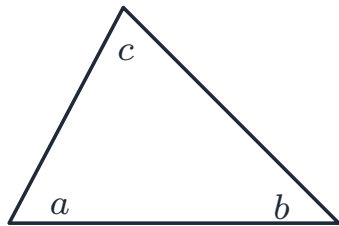
a ball, and half a ball

a frustum

a cone or pyramid with the **top cut off**

A flat side of a solid is a **face** , two faces meet at an **edge** , and the whole outside is its **surface** . Leave construction arcs **visible** – they are marked.

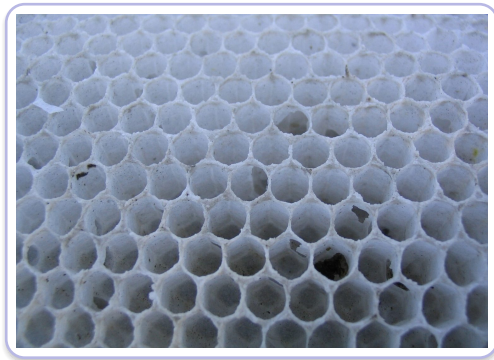
Triangles



$$a + b + c = 180^\circ$$

The three angles of a triangle add up to 180 degrees

Polygons



A honeycomb tessellates the plane with regular hexagons.

The polygons, and the quadrilaterals

By side count

A **pentagon** has 5 sides, a hexagon 6, an **octagon** 8, a **decagon** 10. Each corner is a **vertex**.

Regular polygon

A **regular polygon** has all sides and all angles equal: each exterior angle is $360^\circ/n$, and the interior angle follows.

Parallelogram

A **parallelogram** has two pairs of parallel sides; opposite sides and opposite angles are equal.

Trapezium

A **trapezium** has exactly one pair of parallel sides – which is why its area uses the mean of the two parallel lengths.

Exterior angles of any polygon add to 360° . Start there and the interior angle never has to be memorised.

Similar shapes, continued



Nesting dolls are similar shapes: each one is the same shape as the others, just multiplied by a scale factor

Symmetry



The Taj Mahal has a clear line of symmetry down its centre.

Symmetry, in two dimensions and three

Line symmetry

A shape has a line of symmetry where one half is the mirror image of the other.

Rotational symmetry

The order is how many times it looks the same in one full turn.

Plane of symmetry

In 3D a **plane of symmetry** cuts a solid into mirror-image halves: a **cube** has nine, a **cuboid** three.

Why it helps

A symmetry argument replaces a calculation – it fixes equal lengths and equal angles for free.

Count planes of symmetry by direction: through faces, then through opposite edges. That order stops you missing any.

Exam tips

- Angles on a straight line add to 180° , around a point to 360° , and in a triangle to 180° . Give a **reason** for every step of an angle question.
- Learn the circle theorems: the angle at the centre is *twice* the angle at the circumference; the angle in a semicircle is 90° ; angles in the same segment are equal; opposite angles of a cyclic quadrilateral add to 180° .
- The **exterior angles** of any polygon add to 360° , and each interior angle + its exterior angle = 180° .
- **Bearings** are measured clockwise from north and always written with three figures (e.g. 072°).



Worked example – angles on a straight line

Three angles on a straight line are x , 50° and 70° . Find x .

- Angles on a **straight line** add to 180° .
- $x + 50 + 70 = 180$
- $x + 120 = 180$
- $x = 60^\circ$

Learn the four: straight line 180° , point 360° , triangle 180° , quadrilateral 360° . In an exam, **name the rule** you used – it carries a mark.

Topic 4 – key takeaways

1

Parallel

F equal, Z equal, C add to 180°

2

Polygons

interior sum $(n - 2) \times 180^\circ$

3

Circles

semicircle 90° ; centre = $2 \times$
circumference

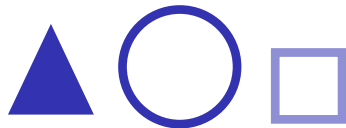
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Similar

area $\times k^2$, volume $\times k^3$

Check yourself

- 1 What do co-interior (C) angles add up to?
- 2 What is each exterior angle of a regular hexagon?
- 3 What is the angle in a semicircle?
- 4 If lengths scale by k , how does volume scale?



Answers 1. 180° 2. 60° 3. 90° 4. by k^3