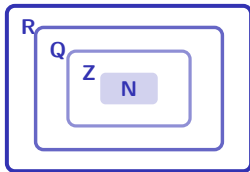


Number

IGCSE Mathematics ■ Topic 1

IGCSE Mathematics

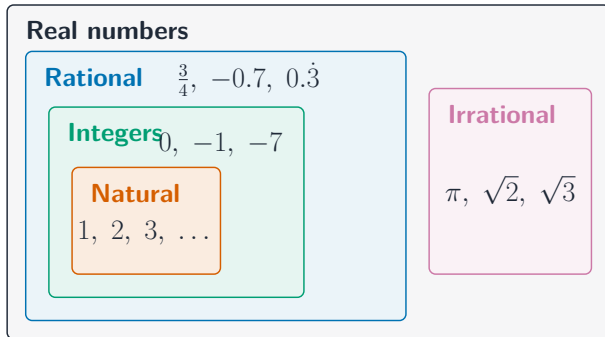


What we will cover

- 1 Types of number
- 2 Primes, HCF & LCM
- 3 Sets & Venn diagrams
- 4 Indices & standard form
- 5 Fractions, decimals & %
- 6 Ratio, rates & bounds



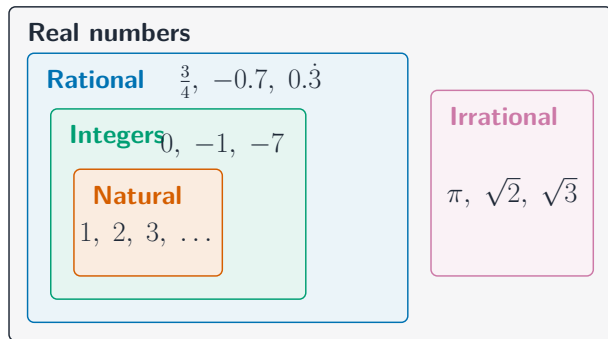
Types of number



- **natural** $1, 2, 3 \dots$;
integers $\dots, -1, 0, 1 \dots$
- **rational** – a fraction $\frac{a}{b}$
- **irrational** – e.g. $\pi, \sqrt{2}$
- **prime** , **square** , **cube**

The number families

- **natural numbers** – the counting numbers $1, 2, 3, \dots$
- **integers** – whole numbers, positive, negative or zero
- **rational** – can be written $\frac{a}{b}$ with a, b integers ($\frac{3}{4}, 5, 0.7$)
- **irrational** – cannot be ($\pi, \sqrt{2}$)
- the **reciprocal** of n is $\frac{1}{n}$: of 4 it is $\frac{1}{4}$, of $\frac{2}{3}$ it is $\frac{3}{2}$



Each set sits inside the next; irrationals are real but not rational

The families of number

Rational and irrational

A **rational number** can be written as a fraction of integers; an **irrational number** cannot – $\sqrt{2}$, π .

Prime number

A **prime number** has exactly two factors, itself and 1. So 1 is not prime, and 2 is the only even prime.

Roots

The **square root** undoes squaring, the **cube root** undoes cubing – and a positive number has two square roots, $\pm$.

Fraction parts

The **numerator** is on top, the denominator below.

Every integer above 1 is a unique **product** of **prime factors**. That single fact is what HCF and LCM are built on.

The words for kinds of number, and the set notation

the number words

natural, integer, rational, irrational, real, prime, square, cube – examiners give marks for using them correctly

a set

a collection of objects; each object in it is an **element**

$\mathcal{E}$

the **universal set** – everything being talked about

$A \subseteq B$

A is a **subset** of B : every element of A is also in B

$A \cap B$ and $A \cup B$

intersection (in both) and union (in either)

A' and $n(A)$

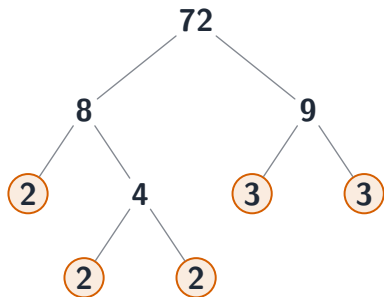
the complement – everything *not* in A – and the number of elements in A

Set notation is examined for its *own* sake, not only inside probability. Learn the symbols as vocabulary.

Factors, multiples, primes, squares and cubes

- A **factor** divides in exactly: the factors of 18 are 1, 2, 3, 6, 9, 18.
 - A **multiple** is that number times an integer: 6, 12, 18, 24, ...
 - A **prime** has **exactly two** factors, 1 and itself: 2, 3, 5, 7, 11, ...
 - **square numbers** : 1, 4, 9, 16, 25, ...
 - **cube numbers** : 1, 8, 27, 64, ...
- 1 is **not** prime – it has only one factor. Examiners ask this.

Primes, HCF and LCM



$$72 = 2^3 \times 3^2$$

HCF & LCM of 72 and 120

$$72 = 2^3 \times 3^2, \quad 120 = 2^3 \times 3 \times 5$$

$$\text{HCF} = \text{lowest powers} = 2^3 \times 3 = \mathbf{24}$$

$$\text{LCM} = \text{highest powers} = 2^3 \times 3^2 \times 5 = \mathbf{360}$$

Keep splitting until every branch ends on a prime (circled); collecting them gives $72 = 2^3 \times 3^2$.

HCF, LCM, and where the remainder goes

Common factor

A **common factor** divides both numbers; the **highest common factor (HCF)** is the largest that does.

Common multiple

A **common multiple** is in both times-tables; the **lowest common multiple (LCM)** is the smallest such.

From prime factors

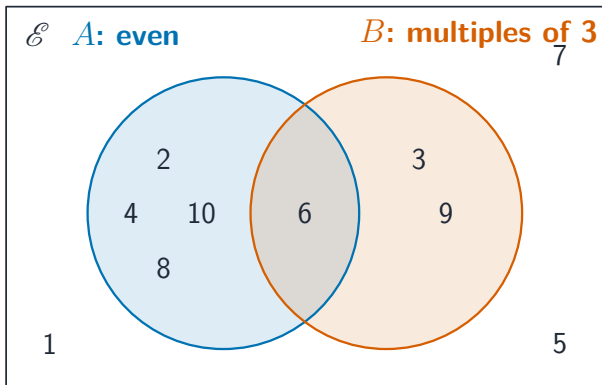
Write both as products of primes: the HCF takes the *lowest* power of each shared prime, the LCM the *highest* power of every prime.

Remainder

A division that does not go exactly leaves a **remainder** – which is exactly what having no common factor means.

HCF for cutting something into equal pieces; LCM for two cycles coming back into step.

Sets and Venn diagrams



A **set** is a collection of **elements** .

- $A \cap B$ – **intersection** (in **both**)
- $A \cup B$ – **union** (in **either**)
- A' – **complement** ; $\emptyset$ – **empty set**

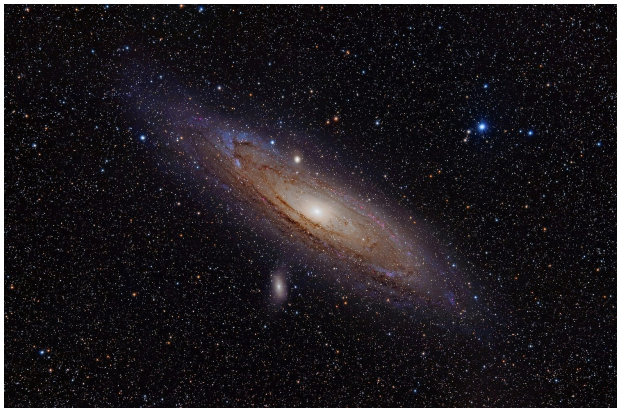


Indices

$$a^m \times a^n = a^{m+n} \quad a^m \div a^n = a^{m-n} \quad (a^m)^n = a^{mn}$$
$$a^0 = 1 \quad a^{-n} = \frac{1}{a^n} \quad a^{1/n} = \sqrt[n]{a}$$

$$\text{e.g. } 2^{-3} \times 2^4 = 2^1 = 2; \quad 81^{1/2} = 9; \quad 8^{-2/3} = \frac{1}{4}.$$

Standard form



Standard form $A \times 10^n$, with
 $1 \leq A < 10$.

■ $4\,500\,000 = 4.5 \times 10^6$

■ $0.00072 = 7.2 \times 10^{-4}$

$(3 \times 10^5)(2 \times 10^{-2}) = 6 \times 10^3$.

Standard form: a number written as $a \times 10^n$

Standard form: $A \times 10^n$, with $1 \leq A < 10$



6 500 000 $\longrightarrow$ 6.5×10^6
point moves 6 places



0.00042 $\longrightarrow$ 4.2×10^{-4}
point moves 4 places

standard form: $a \times 10^n$ with $1 \leq a < 10$

Standard form: a number written as $a \times 10^n$ with $1 \leq a < 10$

Order of operations

work downwards

Brackets



Indices
(powers, roots)



Multiply and Divide
left to right



Add and Subtract
left to right

$$\begin{array}{l} -6 \times -3 + 7 \times 2 \\ \downarrow \text{multiply first: } -6 \times -3 = 18, 7 \times 2 = 14 \\ 18 + 14 \\ \downarrow \text{then add} \\ 32 \end{array}$$

Brackets, then indices, then multiply/divide (left to right), then add/subtract (left to right).

Work out $-6 \times -3 + 7 \times 2$

Multiplications first: $-6 \times -3 = 18$, $7 \times 2 = 14$. Then $18 + 14 = 32$.

The order of operations, applied to the worked example

Negative numbers, and money

- Adding a negative:
 $5 + (-3) = 5 - 3 = 2.$
- Subtracting a negative:
 $5 - (-3) = 5 + 3 = 8.$
- Same signs multiply to a positive, different signs to a negative:
 $-6 \times -3 = 18, -12 \div 4 = -3.$

A change from -5°C to 3°C is a **rise** of 8°C .

Money is ordinary decimals, but answers go to **2 d.p.** – \$4.8 is written \$4.80.

\$1 = €0.92. Convert \$150, then convert back.

$150 \times 0.92 = \text{€}138, \quad 138 \div 0.92 = \$150.$
The **exchange rate** is a **multiplier** one way and a divisor the other.

Using a calculator

- Do not round part-way through a calculation. Keep the full value and round only the final answer.
- Enter time as a decimal of an hour: 2 hours 30 minutes is 2.5 hours, not 2.30.
- Read the display in context: in money, 4.8 means \$4.80; in time, 3.25 hours means 3 hours 15 minutes.

Surds (Extended)

A **surd** is an irrational root, kept in **exact** form:

$$\sqrt{a}\sqrt{b} = \sqrt{ab}, \quad \sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$$

Simplify by taking out the largest **square factor**.

To **rationalise** $\frac{1}{\sqrt{3}-1}$, multiply top and bottom by the **conjugate** $\sqrt{3}+1$ – the surd on the bottom disappears.

Simplify

$$\sqrt{20} = \sqrt{4 \times 5} = 2\sqrt{5}$$

$$\sqrt{200} - \sqrt{32} = 10\sqrt{2} - 4\sqrt{2} = 6\sqrt{2}$$

$$\frac{10}{\sqrt{5}} = \frac{10\sqrt{5}}{5} = 2\sqrt{5}$$

Exponential growth and decay (Extended)

A quantity that changes by the **same percent** each period grows or decays **exponentially** :

$$\text{value} = P \times (\text{multiplier})^t$$

growth $\times (1 + r)$, decay $\times (1 - r)$.

**A \$20 000 car loses 15% a year.
Value after 4 years?**

multiplier = 0.85

$$20\,000 \times 0.85^4 \approx \$10\,440$$

Raise the multiplier to the **power** t – do not subtract 15% four times.

Rounding and estimation

- **decimal places** : digits after the point ($3.14159 \rightarrow 3.14$, 2 d.p.)
- **significant figures** : from the first non-zero digit
- to **estimate** : round each number to 1 s.f., then work it out

$$\text{Estimate } \frac{41.3}{9.79 \times 0.765}$$

$$\approx \frac{40}{10 \times 0.8} = \frac{40}{8} = 5$$

An estimate **checks** a calculator answer – if they are far apart, you slipped.

Rounding, accuracy and surds

Decimal places

Decimal places (d.p.) count digits after the point; round by looking at the next one.

Significant figures

Significant figures (s.f.) start at the first non-zero digit – so 0.00406 to 2 s.f. is 0.0041.

Rationalise the denominator

To **rationalise the denominator**, multiply top and bottom by the surd (or by the conjugate), leaving no root underneath.

Same base

The index laws only combine powers with the **same base** : $a^m \times a^n = a^{m+n}$, and nothing similar exists for different bases.

Round only at the end. Rounding at each step turns a small error into a wrong final answer.



Fractions, decimals and percentages

One value, three forms:

- fraction $\rightarrow$ decimal: divide top by bottom
- decimal $\rightarrow$ % : $\times 100$
- simplify by the **HCF**

$$\frac{3}{8} = 0.375 = 37.5\%$$



fraction

decimal

percentage

Converting between forms

To change	Method	Example
fraction → decimal	divide top by bottom	$\frac{3}{8} = 0.375$
decimal → percentage	multiply by 100	$0.07 = 7\%$
percentage → fraction	put over 100, simplify	$7\% = \frac{7}{100}$
percentage → decimal	divide by 100	$34\% = 0.34$

Simplest form : divide top and bottom by their HCF.
 $\frac{18}{24} = \frac{3}{4}$, both divided by 6.

Percentages and interest



$\% \text{ change} = \frac{\text{change}}{\text{original}} \times 100$. Use
a **multiplier** (+15% $\rightarrow \times 1.15$).

Compound interest

\$500 at 4% for 3 yr $= 500 \times 1.04^3 = \$562.43$

Interest, ratio and rate

interest

money paid for borrowing or for saving; the **principal** is the starting amount

simple interest

a fixed amount each period, always on the original principal

compound interest

$A = P \left(1 + \frac{r}{100} \right)^n$ – interest earns interest, so it grows faster

proportion

two ratios are equal – used for recipes, map **scales** and best value

a rate

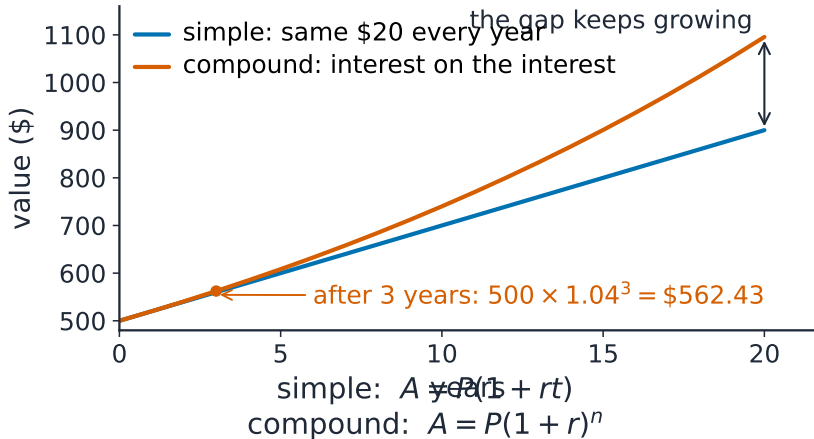
compares two quantities in **different units**: price per kilogram, distance per hour

the standard rates

speed = $\frac{\text{distance}}{\text{time}}$, density = $\frac{\text{mass}}{\text{volume}}$, and **population density**

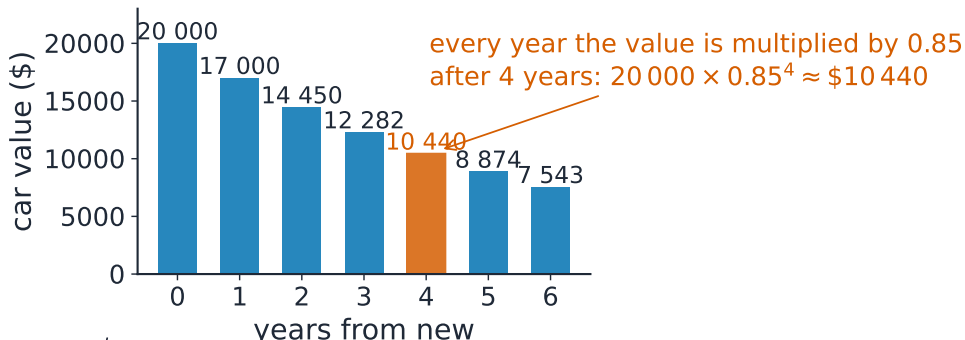
Depreciation is compound interest with a **negative** rate – the same formula, with $1 - r/100$.

Simple against compound, and decay – one



Simple interest is a straight line; compound curves above it

Simple against compound, and decay – two



$V = V_0 r^t$ with $0 < r < 1$ for decay · here $r = 0.85$ each year

A car losing 15% of its current value every year

A car losing 15% of its *current* value every year

Ratio, rates and bounds

ratio

share \$48 as 3:5
→ \$18 & \$30

speed

$$\frac{\text{distance}}{\text{time}};$$
$$\text{density} = \frac{m}{V}$$

bounds

635 m (nearest m):
 $634.5 \leq h < 635.5$

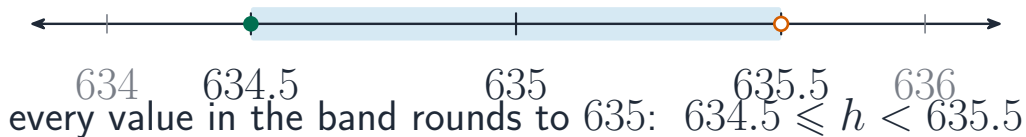
Limits of accuracy

lower bound 634.5

included: it rounds up to 635

upper bound 635.5

not included: it rounds to 636



Everything in the band rounds to 635: the lower bound is included, the upper bound is not

The laws of indices

multiply the powers

$$a^m \times a^n = a^{m+n}$$

add:

$$2^{-3} \times 2^4 = 2^1 = 2$$

divide the powers

$$a^m \div a^n = a^{m-n}$$

subtract:

$$2^3 \div 2^4 = 2^{-1} = \frac{1}{2}$$

power of a power

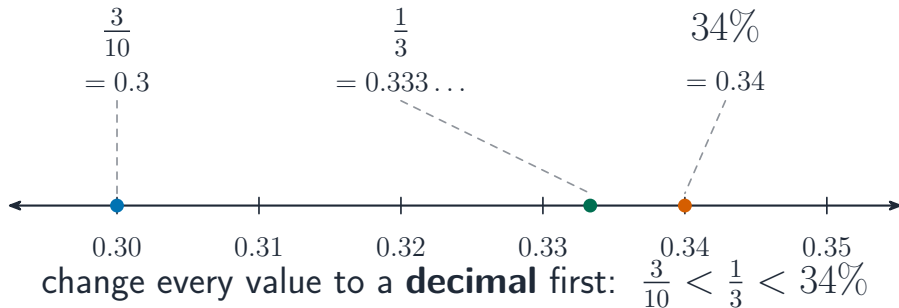
$$(a^m)^n = a^{mn}$$

multiply:

$$(2^3)^2 = 2^6 = 64$$

The three index laws: multiply adds the powers, divide subtracts them, a power of a power multiplies them

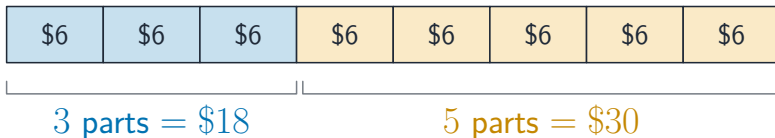
Ordering



Convert every value to a decimal, then place them on a number line

Ratio and proportion

Share \$48 in the ratio 3 : 5 - $3 + 5 = 8$ parts, one part = $48 \div 8 = \$6$



A bar model for sharing 48 dollars in the ratio 3 to 5

Recurring decimals (Extended)

Write $x = 0.1\dot{7}$ as a fraction:

$$100x = 17.\boxed{777\dots}$$

$$10x = 1.\boxed{777\dots}$$

$$90x = 16$$

$$\text{so } x = \frac{16}{90} = \frac{8}{45}$$

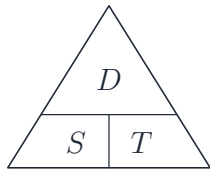
the repeating tails

line up exactly

subtract: the tails **cancel** -
no dots left

Line up the repeating tails, subtract, and the tails cancel

Rates and average speed



cover the one
you want

$$S = \frac{D}{T} \quad (\text{cover } S: D \text{ over } T)$$

$$D = S \times T \quad (\text{cover } D: S \text{ next to } T)$$

$$T = \frac{D}{S} \quad (\text{cover } T: D \text{ over } S)$$

cyclist: 45 km in 3 h 45 min = 3.75 h $\Rightarrow S = \frac{45}{3.75} = 12 \text{ km/h}$

The distance-speed-time triangle: cover the one you want

Working with fractions, order and time

add or subtract

use a **common denominator** – the LCM of the bottoms

multiply

tops together, bottoms together, cancelling first where you can

divide

multiply by the reciprocal – turn the second fraction upside down

ordering by magnitude

$<, \leq, >, \geq$ – and remember a **negative** number is smaller the further it is from zero

special powers

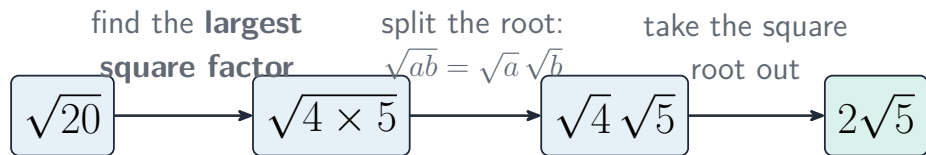
$$a^0 = 1, \quad a^{-n} = \frac{1}{a^n}, \quad a^{1/n} = \sqrt[n]{a}$$

time zones

for timetables and time-zone problems, **add or subtract the difference** between the two places

Convert a mixed number to an improper fraction **before** multiplying or dividing. Trying to do it in place is the standard source of error.

Surds (Extended)



picking a smaller square factor ($\sqrt{20} = \sqrt{2 \times 10}$) gets stuck -
always take out the **largest** square: 4, 9, 16, 25, ...

Simplifying a surd: take out the largest square factor

Exam tips

- Follow **BIDMAS** (brackets, indices, division/multiplication, addition/subtraction) in order, and remember a negative times a negative is positive.
- In **standard form** the number in front is between 1 and 10; a small number (like 0.0004) has a *negative* power of 10.
- A percentage *change* is worked out on the **original** amount. For a reverse percentage, divide by the multiplier (e.g. $\div 1.2$ undoes a 20% rise).
- Do not round part-way through – keep the full value and round only at the end, to the accuracy the question asks for (decimal places or significant figures).
- For **limits of accuracy**, a value rounded to the nearest whole number can be up to 0.5 either side (so 8 means $7.5 \leq x < 8.5$).



Topic 1 – key takeaways

1

HCF & LCM

lowest vs highest powers of the primes

2

Indices

add powers to multiply, subtract to divide

3

Standard form

$A \times 10^n$ with $1 \leq A < 10$

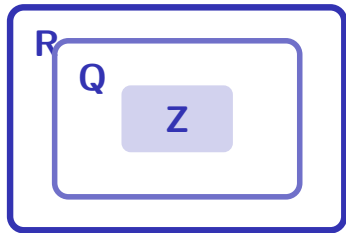
4

Percentages

multiplier; compound = $P(1 + r/100)^t$

Check yourself

- 1 Is $\sqrt{2}$ rational or irrational?
- 2 What is $2^3 \times 2^4$?
- 3 Write 0.00072 in standard form.
- 4 Increase \$40 by 25% using a multiplier.



Answers 1. irrational 2. $2^7 = 128$ 3. 7.2×10^{-4} 4. $40 \times 1.25 = \$50$